Measuring the Discount for Lack of Marketability for Noncontrolling, Nonmarketable Ownership Interests

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Valuation analysts are often asked to value noncontrolling, nonmarketable ownership interests in closely held companies. These valuations may be performed for gift tax, estate tax, generation-skipping transfer tax, income tax, property tax, and other taxation purposes. Depending (1) on the valuation approaches and methods applied and on (2) the benchmark empirical data used, these analyses may initially conclude the valuation of a noncontrolling, marketable ownership interest. In such instances, analysts often have to apply a valuation adjustment to these initial (i.e., marketable) value indications in order to reach the final (i.e., nonmarketable) value conclusion. This discussion summarizes the various factors that analysts typically consider in the measurement of a discount for lack of marketability (DLOM) associated with a noncontrolling, nonmarketable closely held business ownership interest.

INTRODUCTION

Valuation analysts ("analysts") are often asked to value nonmarketable, noncontrolling ownership interests in closely held companies. These valuations may be performed for gift tax, estate tax, generation-skipping transfer tax, income tax, property tax, and other taxation purposes.

Depending on (1) the valuation approaches and methods applied and (2) the benchmark empirical data used, these analyses may initially conclude the valuation of the ownership interest on a noncontrolling, marketable, level of value.

In such instances, analysts often have to apply a valuation adjustment to these initial (i.e., incorrect level of value) value indications in order to reach the final (i.e., correct level of value) value conclusion.

This discussion summarizes the various factors that analysts typically consider in the measurement of a discount for lack of marketability (DLOM)

associated with a noncontrolling, nonmarketable closely held business ownership interest.

The difference in value between a liquid business ownership interest compared to an otherwise comparable illiquid business ownership interest may be substantial. This value difference is often referred to as the DLOM.

This discussion summarizes the following gift-tax-related and estate-tax-related business valuation topics:

1. The concepts of business ownership interest liquidity and illiquidity

2. The various empirical models that analysts often use to estimate the DLOM

3. The application of the DLOM to the valuation of a closely held business ownership interest

4. The factors that influence the magnitude of the DLOM

> **"Typically, there is a spectrum of ownership interest marketability, ranging from fully marketable to fully nonmarketable."**

Liquidity of the Subject Ownership Interest

The terms marketability and liquidity are sometimes used interchangeably. However, there are differences between these two terms.

Barron's Dictionary of Business Terms defines marketability and liquidity as follows:

> **Marketability**. Speed and ease with which a particular security may be bought and sold. A stock that has a large amount of shares outstanding and is actively traded is highly marketable and also liquid. In common use, *marketability* is interchangeable with *liquidity*, but *liquidity* implies the preservation of value when a security is bought or sold.[1]

For purposes of this discussion, the terms *marketability* and *lack of marketability* apply to a fractional ownership interest in a closely held business enterprise. The terms *liquidity* and *lack of liquidity* (or *illiquidity*) apply either to an overall business enterprise or to a controlling ownership interest in the business enterprise.

Typically, the attribute of marketability is not an either/or proposition. There are degrees of marketability. Typically, there is a spectrum of ownership interest marketability, ranging from fully marketable to fully nonmarketable.

An ownership interest of a publicly traded security can typically be converted into cash quickly, at a certain price, and at a low transaction cost. This is the typical benchmark for a fully marketable security.

At the other end of the marketability spectrum is an ownership interest in a closely held business entity that pays no dividends or other distributions, requires capital contributions, and limits ownership of the company to certain individuals.

Common Reasons to Apply a Valuation Adjustment

The population of potential buyers for most closely held company ownership interests is a small percentage of the population of potential buyers for most publicly traded securities.

In fact, typically it is illegal for an individual owner or for a company issuer to sell closely held securities to the general public without first registering the security offering with either the Securities Exchange Commission (SEC) or the state corporation commission.

Such a security offering registration is an expensive and time-consuming process. Furthermore, a noncontrolling stockholder cannot register closely held shares for public trading. Only the company itself can register its securities for public trading.

Besides the problems associated with selling closely held company ownership interests, it is also difficult for investors to hypothecate these securities. The value of closely held company ownership interests is further impaired by the unwillingness of banks and other lending institutions to accept such securities as loan collateral.

Benchmark from Which to Apply the Valuation Adjustment

In the gift tax or estate-tax-related valuation of a closely held company, analysts typically apply some combination of three generally accepted business valuation approaches:

1. Market approach
2. Income approach
3. Asset-based approach

Depending on (1) the individual business valuation variables used and (1) the individual business valuation methods used in the analysis, these three valuation approaches may conclude value indications on either:

1. a controlling ownership interest level of value or
2. a noncontrolling ownership interest level of value.

In the typical application of all three generally accepted business valuation approaches, the resulting value indications are typically concluded on a marketable ownership interest basis.

The magnitude of the specific DLOM depends on the facts and circumstances related to:

1. the subject closely held company and
2. the subject nonmarketable business ownership interest.

This discussion summarizes the factors that analysts typically consider in the measurement of a DLOM.

ANALYTICAL MODELS THAT MAY BE USED TO MEASURE THE DLOM

Analysts often consider two types of models to measure the appropriate level of the DLOM:

1. Empirical models
2. Theoretical models

Generally, the so-called empirical models use analyses that are based on empirical capital market transaction observations—rather than on theoretical economic principles.

Generally, the so-called theoretical models do not rely on actual capital market pricing evidence. Rather, theoretical models are based on fundamental microeconomic relationships.

Empirical Models

Empirical models rely on actual transactional data to provide evidence for estimating the amount of a DLOM.

There are two categories of studies that are often used to measure the DLOM for a noncontrolling ownership interest in a closely held company:

1. Studies of price discounts on the sales of restricted shares of publicly traded companies (i.e., the restricted stock studies)
2. Studies of price discounts on private stock sale transactions prior to an initial public offering (i.e., the pre-IPO studies)

These data are applicable to an initial—or unadjusted—value indication that represents the estimated price at which the subject ownership interest could be sold if it were registered and freely traded in a public stock exchange.

Theoretical Models

Theoretical models do not directly derive DLOM conclusions from transactional data. The theoretical models that may be used to estimate the DLOM for the valuation of a closely held company security generally fall into two categories:

1. Option pricing models (OPM)
2. Discounted cash flow (DCF) models

THE EMPIRICAL MODELS

Restricted Stock Studies

Publicly traded companies often raise capital by completing a private placement of debt or equity securities. In an equity private placement, a company can issue either registered stock to general investors or unregistered (i.e., restricted) stock to an accredited investor.

Registered stock includes the shares of publicly traded companies that generally can be freely traded in the open market. Unregistered shares of stock are not registered for trading on a stock exchange.

When publicly traded companies issue restricted (unregistered) stock, the restricted stock is typically sold at a price discount compared to the price of the (registered) publicly traded stock.

Companies are willing to accept a price discount on the sale of restricted stock. This is because the time and cost of registering the new stock with the SEC would make the stock issuance/capital formation impractical.

These observed price discounts (i.e., public stock price compared to same company private stock price) indicate a DLOM. These stock price discount data are the basis for the restricted stock studies discussed below.

SEC Rule 144[2] governs the purchase and sale of stock issued in unregistered private placements. According to the SEC, "When you acquire restricted securities or hold control securities, you must find an exemption from the SEC's registration requirements to sell them in the marketplace. Rule 144 allows public resale of restricted and control securities if a number of conditions are met."[3]

The conditions mentioned in SEC Rule 144 relate to the following:

1. Investment holding period
2. Adequate current information
3. A trading volume formula
4. Ordinary brokerage transactions
5. Filing of a notice with the SEC

The investment holding period restrictions on the transfer of restricted stock eventually lapse, usually after a period ranging from six months to two years.[4]

At that point, the trading volume formula is typically the most restrictive sale condition of SEC Rule 144. The trading volume formula allows the securities to be "dribbled out" in the marketplace.

Depending on the size of the block of the subject securities, the dribble-out formula may require the investor to sell small portions of the securities over a multiyear period.

Rather than dribble out the sale of the restricted securities, the restricted stock owner can sell the securities in a privately negotiated transaction, subject to the Securities Act of 1933, Section 4(1) and Section 4(2).

Until 1995, restricted stock sale transactions had to be reported to the SEC. Since 1995, analysts have collected restricted stock sale transaction data from private sources.

Therefore, there are data available on the prices of private transactions in restricted securities. These data are sometimes used for comparison with prices of the same company unrestricted securities eligible for trading on the open market.

The conclusions of this restricted stock pricing evidence are discussed in the next section.

Restricted Stock Study Conclusions

Exhibit 1 summarizes 20 restricted stock studies (i.e., 18 total studies, with 2 studies split into 2 subsets) that cover several hundred transactions spanning the late 1960s through 2013.

These studies generally indicate a decrease in the average DLOM after 1990. The restricted stock transactions analyzed in the studies covering the 1968 to 1988 period (where the average indicated DLOM was approximately 35 percent) were generally less marketable than the restricted stocks analyzed after 1990 (where the average indicated DLOM was typically less than 25 percent).

Analysts typically attribute this indicated decrease in price discounts to the following factors:

1. The increase in volume of privately placed stock under SEC Rule 144(a)

2. The change in the minimum SEC-required holding period under Rule 144—from two years to one year—that took place as of April 29, 1997[5]

Increased volume was the result of a Rule 144 amendment in 1990 that allowed qualified institutional investors to trade unregistered securities among themselves. By increasing the potential buyers of restricted securities, the marketability of these securities generally increased.

As it became easier to find a buyer for restricted securities after 1990, the average restricted stock price discount decreased.

The same trend occurred after the SEC-required holding period decreased from two years to one year in 1997.

On December 17, 2007, the SEC issued revisions to Rules 144 and 145.[6]

The revisions included shortening the holding period for restrict-

Exhibit 1
Restricted Stock Studies
Summary of Implied Level of DLOM

Restricted Stock Study	Observation Period of Study	Observed Average or Median Price Discount
SEC Overall Average	1966–69	25.8%
SEC Nonreporting OTC Companies	1966–69	32.6%
Milton Gelman	1968–70	33.0%
Robert R. Trout	1968–72	33.5%
Robert E. Moroney	1969–72	35.6%
J. Michael Maher	1969–73	35.4%
Standard Research Consultants	1978–82	45.0%
Willamette Management Associates	1981–84	31.2%
Hertzel and Smith [a]	1980–87	20.1%
William L. Silber	1981–88	33.8%
Bajaj, Denis, Ferris, and Sarin [b]	1990–95	22.2%
Johnson Study	1991–95	20.0%
Management Planning, Inc.	1980–96	27.0%
FMV Opinions, Inc. [c]	1980–14	19.3%
Greene and Murray	1980-12	24.9%
Columbia Financial Advisors, Inc.	1996–97	21.0%
Columbia Financial Advisors, Inc.	1997–98	13.0%
LiquiStat	2005–06	32.8%
Angrist, Curtis, and Kerrigan	1980–09	15.9%
Stout Risius Ross	2005–10	10.9%

[a] The observed price discount of 20.1 percent represents the overall average private placement discount reported in this study.
[b] This study attributes price discount to factors other than marketability (i.e., compensation for the cost of assessing the quality of the firm and for the anticipated costs of monitoring the future decisions of its managers).
[c] Represents results of the latest published study. The database is routinely updated and available for purchase at www.bvmarketdata.com.

ed securities of issuers that are subject to the Securities Exchange Act of 1934 reporting requirements ("reporting companies") from one year to six months. "Under the amended Rules 144, after six months, if the issuer is a reporting company, . . . nonaffiliates may sell restricted securities without further limitations, including manner-of-sale or volume limitations."[7]

The holding period remains at one year for nonreporting issuers. This amendment became effective February 15, 2008.

It is important for analysts to compare the market for the subject closely held company with the market for restricted securities. If the expected holding period for the closely held company stock is two years or greater, it may be more supportable to select a DLOM based on the restricted stock studies conducted prior to 1990.

Alternatively, if the subject closely held stock is likely to be liquidated within six months or one year, the post-1990 studies may be more meaningful.

Another characteristic of the restricted stock studies is the wide range in price discounts observed within each study. Although the average price discounts calculated in the restricted stock studies are similar, the range of price discounts observed in each study was large, ranging from a price premium to price discounts approaching 90 percent.

One explanation for the wide range in price discounts is the myriad of company-specific and security-specific factors that affect the DLOM. While a DLOM is clearly indicated from the studies, it is up to the analyst to consider how the subject interest relates to the price discounts observed in the restricted stock studies.

Restricted shares of public stock may not (temporarily) be traded directly on a stock exchange. However, in a short time period, the investor has certainty that the trading restrictions will lapse. In contrast, the stock of a closely held company may never be traded on a public stock exchange.

The prospect of any efficient marketability is much lower for closely held company shares compared to restricted public company shares.

Therefore, the appropriate level of the DLOM related to closely held ownership interests may be greater than the price discounts concluded by restricted stock studies.

The Pre-IPO Studies

The second type of empirical analysis is the pre-IPO study. A pre-IPO study examines sale transactions in the stock of a closely held company that has subsequently achieved a successful IPO.

In a pre-IPO study, the DLOM is quantified by analyzing the difference between:

1. the public market price of the IPO and
2. the private transaction price at which a stock was sold prior to the IPO.

The following discussion summarizes three pre-IPO studies.

Emory Studies

A number of studies were conducted under the direction of John D. Emory, currently president of Emory & Co. in Milwaukee, Wisconsin.[8]

These studies covered various time periods from 1980 through 2000.[9]

The various Emory studies excluded the following types of companies:

1. Development stage companies
2. Companies with a history of real operating losses
3. Companies with an IPO price less than $5 per share
4. Foreign companies
5. Banks, saving and loans, real estate investment trusts, and utilities

Except for the 1997 through 2002 study, Emory used the same methodology for the studies. The 1997 through 2002 study focused on sale transactions of common and convertible preferred stock, and did not exclude companies on the basis of financial strength.

The observations in each study consisted of companies with an IPO in which Emory's firm either participated or received a prospectus. The prospectus for each of the 4,088 offerings was analyzed to determine the relationship between:

1. the IPO price and
2. the price at which the latest private transaction took place (up to five months prior to the IPO).

The mean and median price discounts from all of the transactions analyzed in the Emory pre-IPO studies equal 46 percent and 47 percent, respectively.[10] The fact that these price discounts are greater than the restricted stock study price discounts seems reasonable. The pre-IPO stock

sales occurred when there was not an established secondary market for the subject stock.

Exhibit 2 summarizes the results of the Emory studies.

Valuation Advisors Studies

Valuation Advisors, LLC (VA), maintains a database that includes over 3,500 pre-IPO transactions that occurred within two years of an IPO.[11]

These transactions are arranged into five time periods: four 3-month intervals for the 12 months immediately before the IPO, and a single period for the time frame from one to two years before the IPO. The transactions are also arranged by type of security (i.e., stock, convertible preferred stock, or option).

VA performed a pre-IPO study for each year between 1995 and 2012. Exhibit 3 on the following page summarizes the results of the VA studies.

Willamette Management Associates Studies

Willamette Management Associates (WMA) prepared 18 pre-IPO studies covering the period of 1975 through 1997 and an additional study covering the five years 1998 through 2002. As in the previous studies, the 1998–2002 study included only private market stock sale transactions that were considered to be on an arm's-length basis.

The transactional data analyzed in the 1998–2002 WMA pre-IPO study included the following:

1. Sales of closely held stock in private placements

2. Repurchases of treasury stock by the closely held company

All transactions involving the granting of employee, executive, or other compensation-related stock options were eliminated from consideration in the 1998–2002 study. All transactions involving stock sales to corporate insiders or other related parties were eliminated from consideration in the 1998–2002 study.[12]

Due to the small sample size of identified transactions in 2001 and 2002, the data from those years were excluded from the analysis.

The results of the WMA studies are summarized in Exhibit 4. In most cases, the WMA pre-IPO average price discounts were greater than the restricted stock average price discounts.

One explanation for this result is the fact that—unlike pre-IPO transactions—restricted stock transactions involve companies that already have an established public trading market.

Pre-IPO Study Conclusions

The pre-IPO studies cover hundreds of transactions over more than 30 years. Price differences between private transaction prices and public market prices varied under different market conditions, ranging from about 40 to 60 percent (after eliminating the outliers).

Pre-IPO studies provide relevant evidence of the DLOM for privately owned securities. This is because companies in the pre-IPO studies more closely resemble privately held securities to which the DLOM is being applied. The pre-IPO studies are the only DLOM studies that involve transactions in shares of privately owned companies.

THE THEORETICAL MODELS

There are two types of theoretical DLOM measurement models:

1. OPMs

2. DCF models

Exhibit 2
Emory Pre-IPO Studies
Indicated Level of DLOM Results

Pre-IPO Study	Number of Prospectuses Reviewed	Number of Qualifying Transactions	Indicated Price Discount	
			Mean	Median
1980–1981	97	12	59%	68%
1985–1986	130	19	43%	43%
1987–1989	98	21	38%	43%
1989–1990	157	17	46%	40%
1990–1991	266	30	34%	33%
1992–1993	443	49	45%	43%
1994–1995	318	45	45%	47%
1995–1997	732	84	43%	41%
1997–2000 [a]	1,847	266	50%	52%

[a] This is an expanded study. The expanded study focused on sale transactions of common and convertible preferred stock, and did not exclude companies on the basis of their financial strength.
Note: The results above are from "Underlying Data in Excel Spreadsheet for 1980–2000 Pre-IPO Discount Studies, as Adjusted October 10, 2002," located at www.emoryco.com/valuation-studies.shtml.

Exhibit 3
Valuation Advisors Pre-IPO Study
Indicated Median DLOM Results

| IPO Year | Period before IPO in Which Transaction Occurred | | | | | Number of Transactions |
	0–3 Months	4–6 Months	7–9 Months	10–12 Months	1–2 Years	
1995	37.82%	28.62%	60.40%	50.33%	60.64%	34
1996	30.83%	52.97%	56.37%	69.38%	71.81%	270
1997	34.18%	50.00%	67.12%	76.01%	80.00%	212
1998	23.35%	46.67%	68.93%	71.41%	71.91%	212
1999	30.77%	53.89%	75.00%	76.92%	82.00%	694
2000	28.70%	45.08%	61.51%	68.92%	76.64%	653
2001	14.74%	33.17%	33.38%	52.06%	51.61%	115
2002	6.15%	17.33%	21.88%	39.51%	55.00%	81
2003	28.77%	22.30%	38.36%	39.71%	61.37%	123
2004	16.67%	22.68%	40.00%	56.25%	57.86%	334
2005	14.75%	26.10%	41.68%	46.11%	45.45%	296
2006	23.47%	20.69%	40.23%	46.51%	56.27%	264
2007	12.67%	32.55%	43.69%	56.00%	54.17%	459
2008	20.00%	24.21%	45.85%	52.17%	41.18%	41
2009	6.16%	31.85%	26.82%	41.00%	34.87%	108
2010	15.81%	29.89%	44.42%	47.54%	51.88%	358
2011	23.27%	34.62%	43.26%	50.78%	62.10%	281
2012	18.86%	24.07%	28.90%	35.48%	44.78%	292
1995–2012 Average	21.50%	33.15%	46.54%	54.23%	58.86%	
2008–2012 Average	16.82%	28.93%	37.85%	45.39%	46.96%	

Source: Brian K. Pearson. "Valuation Advisors' Lack of Marketability Discount Study™," *Business Valuation Resources Teleconference*, August 23, 2007 (1995–2006); Valuation Advisors database (2007–2012).

Option Pricing Models

OPMs are based on the premise that the cost to purchase a stock option is related to the DLOM. The following discussions summarize four DLOM studies that rely on option-pricing theory.

Chaffe Study

David B.H. Chaffe III authored a 1993 study in which he related the cost to purchase a European put option[13] to the DLOM. Chaffe concluded that "if one holds restricted or non-marketable stock and purchases an option to sell those shares at the free market price, the holder has, in effect, purchased marketability for those shares. The price of that put is the discount for lack of marketability."[14]

Chaffe relied on the Black-Scholes option pricing model to estimate the option price.

The inputs in the Black-Scholes model are as follows:

1. Stock price
2. Strike price
3. Time to expiration
4. Interest rate
5. Volatility

In the Chaffe model, the stock price and strike price equal the marketable value of the private company stock as of the valuation date; the time to expiration equals the time the securities are expected to remain nonmarketable; the interest rate is the cost

Exhibit 4
Willamette Management Associates Pre-IPO Studies
Indicated Level of DLOM Results

Time Period Analyzed	Number of Companies Analyzed	Number of Transactions Analyzed	Standard Mean Price Discount	Trimmed Mean Price Discount [a]	Median Price Discount
1975–78	17	31	34.0%	43.4%	52.5%
1979	9	17	55.6%	56.8%	62.7%
1980–82	58	113	48.0%	51.9%	56.5%
1983	85	214	50.1%	55.2%	60.7%
1984	20	33	43.2%	52.9%	73.1%
1985	18	25	41.3%	47.3%	42.6%
1986	47	74	38.5%	44.7%	47.4%
1987	25	40	36.9%	44.9%	43.8%
1988	13	19	41.5%	42.5%	51.8%
1989	9	19	47.3%	46.9%	50.3%
1990	17	23	30.5%	33.0%	48.5%
1991	27	34	24.2%	28.9%	31.8%
1992	36	75	41.9%	47.0%	51.7%
1993	51	110	46.9%	49.9%	53.3%
1994	31	48	31.9%	38.4%	42.0%
1995	42	66	32.2%	47.4%	58.7%
1996	17	22	31.5%	34.5%	44.3%
1997	34	44	28.4%	30.5%	35.2%
1998	14	21	35.0%	39.8%	49.4%
1999	22	28	26.4%	27.1%	27.7%
2000	13	15	18.0%	22.9%	31.9%

NA = Not applicable

[a] Excludes the highest and lowest deciles of indicated discounts.

Source: Pamela Garland and Ashley Reilly, "Update on the Willamette Management Associates Pre-IPO Discount for Lack of Marketability Study for the Period 1998 Through 2002," *Insights* (Spring 2004).

of capital; and, volatility is a judgmental factor based on volatility of guideline publicly traded stocks.

To apply an OPM to a private company, each of these variables is determined. Some variables, such as the interest rate and strike price, are relatively easy to input. Other variables, such as the holding period and volatility, are more difficult.

According to Chaffe, the volatility for small privately owned companies is likely to be 60 percent or greater. Chaffe reached this conclusion based on the volatility for small public companies that were traded in the over-the-counter market.

According to the study, the appropriate DLOM for a privately held stock with a two-year required holding period and a volatility between 60 percent and 90 percent is between 28 percent and 41 percent.

According to Chaffe, "considering that volatility for shares of most smaller, privately held companies fit the 'VOL 60%-70%-80%-90%' curves, a range of put prices of approximately 28% to 41% of the marketable price is shown at the two-year intercept. At the four-year intercept, these ranges are 32% to 49%,

after which time increases do not substantially change the put price."[15]

Chaffe indicated that his findings were downward biased due to the reliance on European options in the model. Chaffe concluded that his findings should be viewed as a minimum applicable DLOM.

Longstaff Study

Francis A. Longstaff conducted a study that relies on stock options to estimate the DLOM.[16] While Chaffe based his study on avoiding losses, Longstaff based his study on unrealized gains. Another difference between the two studies is that the Longstaff study provides an estimate for the upper limit on the value for marketability.

The Longstaff study is based on the price of a hypothetical "lookback" option.[17]

The Longstaff study assumes an investor has a single-security portfolio, perfect market timing, and trading restrictions that prevent the security from being sold at the optimal time. The value of marketability, based on these assumptions, is the payoff from an option on the maximum value of the security, where the strike price of the option is stochastic.

Exhibit 5 on the next page summarizes the Longstaff study results.

For a five-year holding period and 30 percent standard deviation, the indicated DLOM is over 65 percent. Longstaff analyzed securities with a volatility between 10 percent and 30 percent because "this range of volatility is consistent with typical stock return volatilities."[18]

However, small stocks (such as those traded over the counter and analyzed by Chaffe) typically have greater volatility.

With volatility estimates greater than 50 percent, the Longstaff study indicated DLOM exceeds 100 percent. Some analysts have suggested that the percentage result from the Longstaff model (and other OPMs) is actually a price premium and not a price discount.

Professor Ashok Abbott wrote that, "Often, however, the value of a put option premium, estimating the cost of liquidity, is presented incorrectly as the discount for lack of liquidity. This is similar to the merger premium being treated as a discount for lack of control. Neglecting to convert the option premium to the applicable discount creates the illusion that the estimated discounts are greater than 100%, an impossible solution."[19]

Martin Greene wrote, "Frequently, appraisers compute the option and assume their result is a discount. In reality, the models produce a premium, which must then be converted to a discount."[20]

There is not universal agreement as to whether the OPM analyses produce a price premium or a price discount. Analysts who rely on the OPM analyses should consider how to use the studies to estimate the DLOM.

Finnerty Study

John D. Finnerty conducted an option-pricing study that "tests the relative importance of transfer restrictions on the one hand and information and equity ownership concentration effects on the other in explaining private placement discounts."[21]

The Finnerty option-pricing study is an extension of the Longstaff study. Unlike Longstaff, Finnerty did not assume that investors have perfect market timing ability. Instead, Finnerty modeled the DLOM as the value of an average strike put option.

In addition to analyzing stock options, Finnerty analyzed 101 restricted stock private placements that occurred between January 1, 1991, and February 3, 1997. The Finnerty private placement study concluded price discounts of 20.13 percent and 18.41 percent for the day prior to the private placement and for 10 days prior to the private placement, respectively.

With regard to his option-pricing study, Finnerty concluded that his model:

> calculates transferability discounts that are consistent with the range of discounts observed empirically in letter-stock private placements for common stocks with volatilities between $\delta = 30$ percent and $\delta = 70$ percent but the implied discounts are greater than (less than) those predicted by the model for lower (higher) volatilities.[22]

Finnerty reported the following observations about the importance of dividends, volatility, and the DLOM:

> My model implies that when the stock price volatility is under 30 percent, the appropri-

Exhibit 5			
Longstaff Study			
Upper Bounds for the Level of DLOM Percentage			
Marketability Restriction Period	Standard Deviation = 10%	Standard Deviation = 20%	Standard Deviation = 30%
1 Day	0.421	0.844	1.268
5 Days	0.944	1.894	2.852
10 Days	1.337	2.688	4.052
20 Days	1.894	3.817	5.768
30 Days	2.324	4.691	7.100
60 Days	3.299	6.683	10.153
90 Days	4.052	8.232	12.542
180 Days	5.768	11.793	18.082
1 Year	8.232	16.984	26.276
2 Years	11.793	24.643	38.605
5 Years	19.128	40.979	65.772

ate discount is smaller than the customary discount range of about 25 percent to 35 percent. For example, when δ is between 20 percent and 30 percent and there is a two-year restriction period, the proper discount is in the range from 15.76 percent to 20.12 percent for a non-dividend-paying stock and in the range from 11.50 percent to 15.96 percent for a stock yielding 3.0 percent. The halving of the initial restriction period under Rule 144 since February 1997 has roughly halved the transferability discount.[23]

Long-Term Equity Anticipation Securities (LEAPS) Studies

In September 2003, Robert Trout published a study analyzing LEAPS and the DLOM.[24]

Ronald Seaman updated the Trout LEAPS study several times—the most recent update was in September 2013.[25]

Each of these LEAPS studies was conducted using a similar research logic and research design. The following discussion summarizes these studies.

A long-term equity anticipation security is essentially a long-term stock option that offers price protection for up to two years into the future. Therefore, an investor who desires protection against stock price declines can purchase a LEAPS put option.

The LEAPS studies examined the cost of buying LEAPS put options and concluded that the cost of the LEAPS put option divided by the stock price indicates the DLOM.

Trout examined nine LEAPS as of March 2003 with options expiring January 2005. The nine LEAPS

were for large companies with actively traded securities.[26]

According to Trout, "The data concerning the relative cost of puts as an insurance premium indicate an insurance premium cost equal to about 24 percent of the price. This finding suggests that the minimum discount that one should assign for the lack of marketability of holding privately held stock is at least 24 percent."[27]

The 2013 Seaman study updated and extended the Trout study through November 2012. The Seaman study considered the relationship between the price of the LEAPS (i.e., the price discount) and the following variables:

1. Company size
2. Company risk
3. Latest year profit margins
4. Latest year return on equity
5. Company industry

The Seaman study concluded the following:

1. Company size: Revenue size has a major effect on the cost of price protection with smaller levels of revenue associated with larger discounts.
2. Company risk: Company risk has a large effect on discounts, with higher risk companies, as measured by a company's beta, associated with a larger discount.
3. Latest year profit margin: Company profitability has a mild (but not a major) effect on marketability discounts.
4. Return on equity: The company's latest year return on equity has some effect on discounts particularly at the lower end of returns. For positive returns on equity, there is a minor effect on discounts.
5. Industry: The size of the discount varies by industry, but the discounts vary even more by the individual company.[28]

The Seaman study presented the following observation with regard to the cost of price protection:

[T]he costs of price protection are not constant but vary significantly over time. Economic conditions in November 2008 (recession) caused discounts to double or more over the August 2006 period. By November 2009 economic conditions had moderated. The costs of price protection had gone down by about one-third but were still from 30% to 50% above August 2006 levels.[29]

The LEAPS studies concluded that the observed DLOMs are appropriately viewed as benchmark minimum price discounts when applied to the valuation of privately held companies.

This LEAPS study conclusion is based on the following observations:

1. The underlying securities on which the LEAPS were based are often much larger than the privately held subject company.
2. The underlying securities on which the LEAPS were based are marketable.
3. The LEAPS themselves can be sold at any time during the holding period.
4. There is a known liquidity event (i.e., the sale of the underlying security) for LEAPS.

Option Pricing Model Study Conclusions

The OPM studies discussed above indicate similar price discounts to the empirical studies discussed previously. In the Chaffe, Longstaff, and Finnerty studies, the appropriate DLOM for a privately held company (given certain volatility assumptions) reaches 65 percent.

In the LEAPS studies, the price discount is much lower, but the authors conclude that the indicated price discount represents a minimum DLOM.

Because of their nature, OPM studies generally only consider the factors that affect option pricing: holding period and volatility. Although other factors are considered in the OPMs, the holding period and the volatility factors have the greatest impact on the option prices.

Therefore, OPM studies may understate the measurement of the DLOM. This is because OPM studies ignore other factors that may reduce the marketability for privately held securities (e.g., contractual transferability restrictions).

Basing the size of the DLOM on the two OPM factors appears reasonable. The holding period relates to the duration of time restricted stock must be held and risk relates to volatility. As the restricted stock studies indicate, the longer the required holding period, the greater the price discount a buyer expects.

Volatility is directly related to the DLOM. When an investor owns a security that is restricted from trading, that investor assumes the risk of:

1. not being able to sell the investment if the value begins to decline and
2. not being able to sell the investment to reallocate funds to another investment.

The first risk factor is affected by highly volatile stocks. As volatility increases, the risk of stock price

depreciation increases. As volatility increases, the risk related to holding a nonmarketable security likewise increases.

Due to these factors, the OPM studies provide a general methodology for analyzing the DLOM. These option pricing studies make several contributions to the empirical research referenced above.

The Discounted Cash Flow Models

The DCF method is based on the principle that value equals the present value of future income.

Z. Christopher Mercer and Travis W. Harms described how the DCF model relates to the DLOM:

> Quantitative analyses therefore estimates the value of illiquid interests based on the expectation of benefits (distributions or dividends and proceeds of ultimate sales) over relevant expected holding periods using appropriate discount rates to equate with present values. The process of doing this analysis, in the context of valuing a business at the marketable minority interest level, determines the applicable marketability discount.[30]

The following discussion summarizes two studies that rely on the DCF method.

The Quantitative Marketability Discount Model (QMDM)

Developed by Z. Christopher Mercer, the QMDM is a shareholder-level DCF model that uses a quantitative analysis to calculate the DLOM.

The QMDM calculates the DLOM based on:

1. the expected growth rate in the subject company value,
2. the expected interim cash flow,
3. the expected holding period, and
4. the required holding period return.

In the book, *Quantifying Marketability Discounts*,[31] Mercer provides guidance with regard to estimating these four factors.

In the application of the QMDM, the analyst values the subject company at the entity level, resulting in a valuation as if the security was readily marketable. Next, the analyst estimates shareholder value. The shareholder value represents the nonmarketable value of the subject security.

To calculate the shareholder value, the analyst increases the value of the subject company by the growth rate during the expected holding period.

Next, the analyst discounts the future company value using the required holding period return. Then, the analyst adds the present value of the dividend stream received during the holding period to this present value.

The resulting value equals the shareholder value. The calculation of one minus the ratio of shareholder value to enterprise value equals the DLOM.

The DLOM measured using the QMDM model is highly subject to the model inputs. In the *Estate of Weinberg v. Commissioner*, the Tax Court noted that, "slight variations in the assumptions used in the model produce dramatic differences in the results."[32]

In the *Estate of Janda v. Commissioner*, the Tax Court was concerned with the magnitude of the DLOM calculated using the QMDM model. The Tax Court noted, "We have grave doubts about the reliability of the QMDM model to produce reasonable discounts, given the generated discount of over 65%."[33]

Tabak Model

The Tabak model is a DCF model used to estimate the DLOM based on the capital asset pricing model (CAPM).

The Tabak model "focuses on the extra risks imposed on the owner of a security or interest in a business enterprise, and not on the lack of access to capital. In brief, the theory uses market data on the additional return that investors require in order to hold a risky asset, measured by the equity risk premium, to extrapolate the extra return that the holder of an illiquid asset would require."[34]

Discounted Cash Flow Model Conclusions

The DCF models provide an analysis regarding the cause and the measurement of the DLOM. The QMDM results are sensitive to the model inputs. In addition, the model inputs used in the QMDM and the Tabak model require the application of the analyst's judgment

CONSIDERATION OF OWNERSHIP-INTEREST-SPECIFIC TRANSFERABILITY RESTRICTIONS

The restricted stock studies presented in this discussion present a multitude of factors that affect the DLOM for privately owned companies. Certain factors that affect the DLOM appear frequently.

For example, many of the restricted stock studies indicate that company size, block size, and dividends affect the DLOM.

There are other factors that affect privately owned companies that are not measurable in the restricted stock studies. These factors include contractual restrictions, such as a shareholder agreement, right of first refusal, buy-sell agreement, and the like.

Contractual restrictions can severely limit the marketability of a noncontrolling ownership interest in a privately owned company.

The following list presents some of the contractual restrictions that may affect the DLOM:

1. Buy-sell agreements
2. Shareholder or partnership agreements
3. Rights of first refusal
4. Other contractual transferability restrictions

The more restrictive the agreement or provision, the greater the appropriate DLOM, all else equal.

Other Factors Commonly Affecting the DLOM Measurement

A security is not either marketable or nonmarketable. Rather, there are varying degrees of marketability. The studies discussed above describe a starting point to estimate the DLOM. However, the facts and circumstances of each analysis determine the appropriate DLOM.

It is a matter of analyst judgment to select a DLOM based on:

1. the empirical DLOM evidence,
2. the theoretical DLOM evidence, and
3. the facts and circumstances of each analysis.

The following discussion considers the subject-specific factors that affect the DLOM.

In *Mandelbaum v. Commissioner*,[35] Judge David Laro cited nine specific (but nonexclusive) factors for analysts to consider in developing a DLOM:

1. Financial statement analysis
2. Dividend history and policy
3. Nature of the company, its history, its position in the industry, and its economic outlook
4. The company management
5. The amount of control in the transferred shares
6. The restrictions on transferability

7. The holding period for the stock
8. Subject company's redemption policy
9. Costs associated with a public offering

The *Mandelbaum* decision is cited frequently in decisions related to the measurement of the DLOM. The *Mandelbaum* factors are intuitive, and they reconcile with the empirical studies discussed above.

Analyses of the *Mandelbaum* factors, the empirical studies, the theoretical studies, and other DLOM literature make it clear that many company-specific and security-specific factors affect the magnitude of the DLOM.

These factors generally fall into three categories:

1. Dividend payments
2. Expected holding period
3. Subject company risk

The following discussion summarizes these three categories of DLOM factors.

Dividend Payments

The text *Valuing a Business*[36] explains the importance of dividends:

> Stocks with no or low dividends suffer more from lack of marketability than stocks with high dividends. Besides being empirically demonstrable, this makes common sense. If the stock pays no dividend, the holder is dependent *entirely* on some future ability to sell the stock to realize any return. The higher the dividend, the greater the return the holder realizes without regard for sale of the stock.

An investor in a closely held company would generally prefer some dividends to no dividends. When the subject is a noncontrolling ownership interest, the analyst should also consider that the future dividends may not equal the historical dividends.

Let's assume a closely held company makes an annual dividend payment equal to 100 percent of its annual cash flow. And, let's assume that all company shareholders are related. Under the fair market value standard of value, the willing buyer of a noncontrolling interest in this company will not be a family member.

In order for the economic benefits to remain within the controlling family, the company may:

1. discontinue paying dividends and
2. allocate the cash previously used for dividends to family members.

In this example, the presence of historical dividends is not the only factor to consider when analyzing dividends relative to a private company. The expected future dividends of the company may be considered in the DLOM measurement.

Expected Investment Holding Period

The second factor that affects the DLOM is the expected holding period. Both the *Mandelbaum* decision and Revenue Ruling 77-287[37] state that the expected holding period affects the DLOM. The restricted stock studies, the pre-IPO studies, the OPM studies, and the DCF models all consider holding period as a factor.

This holding period factor is associated with the DLOM for the following reasons:

1. It is clearly measured in empirical studies.
2. It is intuitive.
3. It encompasses a variety of other factors.

In Exhibit 6, the size of the DLOM is related to the expected holding period. As the holding period increases, so does the DLOM.

Closely Held Company Risk

The third factor that affects the DLOM is the subject closely held company risk. The restricted stock studies and the OPM studies conclude that the size of the DLOM is related to the stock price volatility (one measure for risk). The studies also associate company size (another measure for risk) with the DLOM size.

For example, the McConaughy, Cary, and Chen restricted stock study indicates, "There are three factors that remain significant: size, stability of revenue growth, and stock price volatility. These three factors clearly reflect the riskiness of investing in a company."[38]

Each of these three factors relates to the subject closely held company risk.

A large company is a "safer" investment than a similar small company, all other factors being equal. This conclusion is illustrated by comparing the expected rates of return on large-capitalization companies to small-capitalization companies. Ibbotson Associates makes this comparison:

> One of the most remarkable discoveries of modern finance is the finding of a relationship between company size and return. . . . The relationship between company size and return cuts across the entire size spectrum. . . . Small-cap stocks are still considered riskier investments than large-cap stocks.

Exhibit 6
Emory Studies for 1980 to 2000 (after a 2002 Revision)
Price Discounts versus Time between Transaction and IPO

Number of Days	Price Discount Average	Price Discount Median	Transaction Count
0–30	30%	25%	18
31–60	40%	38%	72
61–90	42%	43%	162
91–120	49%	50%	161
121–153	55%	54%	130
Total			543

Source: Institute of Business Appraisers Annual National Conference, June 2, 2003.

Investors require an additional reward, in the form of additional return, to take on the added risk of an investment in small-cap stocks.[39]

Large closely held companies are perceived as safer investments than are small closely held companies. Larger earnings typically enable a closely held company to:

1. withstand downturns in the economy and subject industry and
2. capitalize on growth opportunities.

Factors in addition to size can also affect the subject company risk. The following list includes some of the common factors that may affect the subject closely held company risk:

- Historical financial ratios
- Historical earnings trends/volatility
- Management depth
- Product line diversification
- Geographic diversification
- Market share
- Supplier dependence
- Customer dependence
- Deferred expenditures
- Lack of access to capital markets

SUMMARY AND CONCLUSION

The DLOM Adjustment

Analysts are often asked to value noncontrolling, nonmarketable ownership interests in closely held companies. These valuations may be performed for gift tax, estate tax, generation-skipping transfer tax, income tax, property tax, and other taxation purposes.

Depending on the valuation approaches and methods applied and on the benchmark empirical data used, the analyses may initially conclude the valuation of a noncontrolling, marketable ownership interest. In such initial value instances, analysts often have to apply a valuation adjustment in order to reach the final (i.e., correct level of value) value conclusion.

This discussion summarizes the various factors that analysts typically consider in the measurement of a discount for lack of marketability (DLOM) associated with a noncontrolling, nonmarketable closely held business ownership interest.

The Application of the DLOM in the Valuation

In measuring the DLOM, analysts may consider all of the facts and circumstances relevant to the subject business ownership interest.

Based on the facts of a specific analysis, there are times when one study is more relevant than another. This is because marketability and lack of marketability are relative (and not absolute) terms.

The restricted stock studies conducted prior to 1990 indicated a DLOM of around 35 percent. After 1990, the DLOM indicated in the restricted stock studies decreased to around 25 percent. The average DLOM indicated in the pre-IPO studies was approximately 45 percent to 50 percent.

The different degrees of marketability in the ownership interests that supply the data points used in the various DLOM studies is a reason for the different DLOM indications.

If the subject closely held company or ownership interest has an expected holding period of one year or less, it may be appropriate to place more emphasis on the DLOM results from the post-1990 restricted stock studies than the pre-IPO studies.

If a liquidity event for the subject closely held company or ownership interest is not expected to occur for many years, then the results from pre-IPO DLOM studies may be more meaningful.

In addition to comparing the subject business ownership interest to the published DLOM studies, the subject ownership interest may require an upward or downward adjustment relative to the selected benchmark.

Some closely held company-specific and ownership-interest-specific factors include the following:

1. Historical and expected dividend payments
2. The expected holding period
3. Subject closely held company risk

Notes:

1. John Downs and Jordan Elliot Goodman, eds., *Barron's Dictionary of Finance and Investment Terms*, 6th ed. (Hauppauge, NY: Barron's, 2003), 406.

2. 17 CFR 230.144 (revised April 1, 1990).

3. SEC website: http://www.sec.gov/investor/pubs/rule144.htm

4. On February 18, 1997, the SEC adopted amendments to reduce the holding period requirements under Rule 144 of the Securities Act from two years to one year for the resale of limited amounts of restricted securities (the amendment became effective April 29, 1997). Further, on November 15, 2007, the SEC adopted similar amendments which reduced the holding period requirements from one year to six months (effective February 15, 2008).

5. See, for example, Bruce Johnson, "Restricted Stock Discounts, 1991–95," *Shannon Pratt's Business Valuation Update* (March 1999); Rod Burkert, "Cure for Declining Discounts, Deconstruct the Studies," *Trusts & Estates* (March 2004); and Robert Reilly, "Willamette Management Associates' Discount for Lack of Marketability Study for Marital Dissolution Valuations," *American Journal of Family Law* (Spring 2005).

6. 17 CFR Parts 230 and 239, December 17, 2007.

7. John A. Menicucci Jr., "SEC Adopts Amendments to Rule 144 & Rule 145," *The Nebraska Lawyer* (April 2008).

8. Emory was formerly with Robert W. Baird & Co. where the studies prior to April 1997 were conducted.

9. John D. Emory, "The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock—January 1980 through June 1981," *Business Valuation News* (September 1985): 21–24, also in *ASA Valuation* (June 1986): 62–66; "The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock, January 1985 through June 1986," *Business Valuation Review* (December 1986): 12–15; "The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock (August 1987–January 1989)," *Business Valuation Review* (June 1989): 55–57; "The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock, February 1989–July 1990," *Business Valuation Review* (December 1990): 114–16; "The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock, August 1990 through January 1992," *Business Valuation Review* (December 1992): 208–212; "The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock, February 1992 through July 1993," *Business Valuation Review* (March 1994): 3–5; "The Value of Marketability as Illustrated in Initial Public Offerings of Common

Stock, January 1994 through June 1995," *Business Valuation Review* (December 1995): 155–160; "The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock, November 1995 through April 1997," *Business Valuation Review* (September 1997): 123–131; John D. Emory Sr., F.R. Dengel III, and John D. Emory Jr., "The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock, May 1997 through December 2000, *Business Valuation Review* (September 2001): 15–19; and "Underlying Data in Excel Spreadsheet for 1980-2000 Pre-IPO Discount Studies, as Adjusted October 10, 2002," located at http://www.emorybizval.com/valuation-studies.shtml.

10. See John D. Emory Sr., F.R. Dengel III, and John D. Emory Jr., "Discounts for Lack of Marketability: Emory Pre-IPO Discount Studies 1980–2000, as Adjusted October 10, 2002," www.emoryco.com/valuation-studies.shtml.

11. The database is available on a subscription basis from www.bvmarketdata.com.

12. The specific analytical procedures performed in the various WMA pre-IPO DLOM studies are detailed in Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. (New York: McGraw-Hill, 2000), 408–411.

13. European options have a single exercise date. In contrast, the holder of an American option can exercise the option at any time during the existence of the option.

14. David B.H. Chaffe III, "Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations," *Business Valuation Review* (December 1993): 182–6.

15. Ibid.: 184.

16. Francis A. Longstaff. "How Much Can Marketability Affect Security Values?" *The Journal of Finance* (December 1995): 1767–74.

17. A "lookback" option differs from most other options in that the holder can look back at the end of the option's life and retroactively exercise the option at either the lowest stock price (for a call option) during the holding period or the highest stock price (for a put option) during the holding period.

18. Longstaff, "How Much Can Marketability Affect Security Values?": 1771.

19. Ashok Abbott, "Discounts for Lack of Liquidity: Understanding and Interpreting Option Models," *Business Valuation Review* 28, no. 3 (Fall 2009): 145.

20. Martin Greene, "Do Maximum Strike Price Lookback (Longstaff) and Other Put Option Models Produce a Marketability Premium or a Discount?" *Business Valuation Update* (October 2010): 26.

21. John D. Finnerty, "The Impact of Transfer Restrictions on Stock Prices," *Analysis Group/Economics* (October 2002).

22. Ibid.: 28–29.

23. Ibid: 30

24. Robert R. Trout, "Minimum Marketability Discounts," *Business Valuation Review* (September 2003).

25. Robert M. Seaman, "Latest LEAPS Study Sheds Light on Company Size and DLOMs," *Business Valuation Update* 19, no. 9 (September 2013).

26. Companies examined included Amazon, Ford Motor, General Motors, Morgan Stanley, Microsoft, Nextel, Qlogic, Qualcom, and Tyco.

27. Trout, "Minimum Marketability Discounts": 124–5.

28. Seaman, "Minimum Marketability Discounts—5th Edition," March 2010.

29. Seaman, "Minimum Marketability Discounts—4th Edition," whitepaper issued March 2009: 14.

30. Z. Christopher Mercer and Travis W. Harms, "Marketability Discount Analysis at a Fork in the Road," *Business Valuation Review* (December 2001): 23.

31. Z. Christopher Mercer, *Quantifying Marketability Discounts* (Memphis: Peabody Publishing, 1997).

32. Estate of Weinberg v. Commissioner, T.C. Memo 2000-51 (February 15, 2000).

33. Donald J. Janda and Dorothy M. Janda v. Commissioner, T.C. Memo 2001-24 (February 2, 1001).

34. David Tabak, "A CAPM-Based Approach to Calculating Illiquidity Discounts," NERA Economic Consulting publication (November 11, 2002), www.nera.com.

35. Mandelbaum v. Commissioner, T.C. Memo 1995-255 (June 13, 1995).

36. Shannon P. Pratt, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 5th edition, (New York: McGraw-Hill, 2008), 495.

37. According to Rev. Rul. 77-287, Section 6.02, "the longer the buyer of the shares must wait to liquidate the shares, the greater the discount."

38. Daniel L. McConaughy, David Cary, and Chao Chen, "Factors Affecting Discounts on Restricted Stock," *Valuation Strategies* (November/December 2000): 46.

39. *Ibbotson SBBI 2015 Classic Yearbook* (Chicago: Morningstar, 2015), 99, 113.

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