Best Practices

A Closer Look at Volatility in Stock Option Valuations

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Volatility is considered by many valuation analysts to be an important input in performing stock option valuations. Estimating volatility can be complex. This is because the analysis involves perspectives that at the same time (1) may result in different or conflicting indications and (2) may be interrelated and overlapping. This discussion summarizes three perspectives the analyst may consider in estimating volatility and presents insights and guidelines that may aid the analyst through the analysis.

Introduction

The value of a stock option, regardless of the valuation methodology that is applied, is most sensitive to the analyst's estimate of the range of the expected trading price of the underlying stock. In stock option valuation models, the range of the expected trading price of the underlying stock is called its volatility.

A stock option is more valuable when there is a wide range of expected trading prices for the underlying stock (holding all other variables constant). A stock option is less valuable when there is a narrow range of expected stock trading prices.

This is reasonable because the holder of a call option has the right, but not the obligation, to buy the underlying stock at a price that has already been fixed (the strike price). When the underlying stock might trade within a wide range in the future (before the expiration date of the option), the call option holder has more reason to expect to be able to sell the underlying stock (after exercising the option) at a premium above the strike price.

There is no one generally accepted method to estimate the volatility of the expected trading price of the stock that underlies a stock option.

The factors that analysts consider when estimating volatility for stock option valuation purposes are the subject of this discussion.

To illustrate the sensitivity of a stock option's value to these factors, we will consider whether economic damages have been incurred by an employee when the employer unilaterally reduced the expiration date of the employee's stock options in a hypothetical example.

The valuation analyst may consider several perspectives when estimating volatility as an input in valuing a stock option.

Different perspectives for estimating volatility may sometimes result in widely disparate volatility indications. A small change in the volatility assumption often results in a significant change in the stock option value.

The selection of the best perspective is seldom a simple one. In order to determine the best perspective, the analyst should consider taking several perspectives and then examining each of the perspective results. The selected volatility should be (1) supportable, (2) consistent with the purpose of the valuation assignment, and (3) explainable and understandable.

We discuss three perspectives to consider in estimating volatility as an input in valuing stock options. These three perspectives are as follows:

1. Historical volatility versus implied volatility
2. Time horizon over which volatility is measured

3. The changes in volatility during different periods of time in the past

The interrelationships of these perspectives and their overlapping combinations add to the complexity of the volatility selection.

A Brief Review of Stock Option Valuation Methodology

While the focus of this discussion is on volatility and its impact on the value of stock options, we will also review the inputs into the Black-Scholes-Merton ("BSM") model and indicate how changes to each input affects the stock option value.

The inputs into the BSM model are the (1) current stock price, (2) strike price, (3) term to expiration, (4) risk-free rate, (5) dividend yield, and (6) volatility.

Holding all other inputs constant, the value of a stock option will:

1. Increase with an increase in the current stock price
2. Decrease with an increase in the strike price
3. Increase with an increase in the term to expiration
4. Increase with an increase in the risk-free rate
5. Decrease with an increase in the dividend yield
6. Increase with an increase in the volatility

Historical Volatility versus Implied Volatility

Volatility is commonly measured in two primary ways.

The first way is by examining historical stock prices. Historical volatility is, as the name suggests, backward-looking and examines the changes in the observable stock price. When the analysis relies on historical, observable stock prices it means that this is the best estimate of future stock prices. Historical volatility is a statistical measure of the dispersion of returns for a given stock. It is the relative rate at which the price of a stock moves up or down. It is measured by calculating the standard deviation of changes in the price of a stock.

The second way to measure volatility is by examining implied volatility.

This involves examining the stock price volatility that is "implied" by an observable stock option's trading price and solving for the volatility that is implied by the trading price of that option compared to the trading price of the underlying stock.

While historical volatility is backward-looking, implied volatility is forward-looking. It is the market's expectation of the future volatility of the stock.

While historical volatility can be calculated directly, implied volatility cannot. Instead, implied volatility of a stock may only be calculated indirectly by examining the observable trading price of an option on that underlying stock.

Generally, option traders evaluate stock options and execute their trades based on implied volatility and not on the actual stock option premium price. "Traders often quote the implied volatility of an option rather than its price. This is convenient because the implied volatility tends to be less variable than the option price."

Implied volatility is calculated by entering all the inputs into the BSM model except for volatility and then solving for the volatility that makes the stock option price generated from the BSM model equal to the actual trading price of the stock option.

The implied volatility for a particular company's stock calculated this way is often different from the volatility calculated by relying on the historical trading price of the underlying shares.

If implied volatility is lower than historical volatility, then an investor may consider the observed stock option trading price to be a good investment. In other words, if the higher historical volatility is applied in the BSM model, then the resulting stock option price will be higher than the observable option trading price and the observable option trading price would appear to be undervalued. Similarly, if implied volatility is higher than historical volatility, this stock option price resulting from the BSM model will be lower than the observable option trading price which may make an investment in that stock option appear to be less attractive.

Analysts typically rely on historical volatility when valuing stock options. Since implied volatility captures the market's expectations for future volatility and, since valuations using options are typically forward-looking, it may be reasonable to consider the implied volatility assumptions in the valuation analysis.

"Implied volatilities can be used to monitor the market's opinion about the volatility of a particular stock. Whereas historical volatilities are "backward looking," implied volatilities are "forward looking."

It is therefore not surprising that predictions of a stock's future volatility based on implied volatilities tend to be slightly better than those based on historical volatilities."

Let's consider an example to see how historical volatility differs from implied volatility and determine whether there are any meaningful value implications related to the difference.

We examined an option of a hypothetical publicly traded company, ABC Company (ABC). An executive of ABC is granted an option to buy 100,000 shares of ABC with a strike price of $38. On July 1, 2016, ABC notified the executive that the exercise period for the option was going to be reduced by one year from an expiration date in 1.5 years, to a shorter expiration date of 0.5 years. The executive wants to understand how the value of his option has changed and whether the shortened expiration has damaged him economically. The details of the stock option and the BSM model inputs are presented in Exhibit 1.

Historical Volatility

We first examine the impact of a shorter expiration period by applying the BSM model after selecting volatility based on the historical changes in the observable publicly traded stock price.

Based on the inputs in Exhibit 1, we calculate the value of the option with the original expiration term by applying the BSM model relying on historical volatility assumptions for an option with a remaining term of 1.5 years.

Next, we calculate the value of the option with the shorter expiration term based on historical volatility for an option with remaining term of 0.5 years. The difference in the two indications of value is an indication of the economic damages suffered by the executive that is caused by the expiration term being reduced by one year. The results of these calculations are presented in Exhibit 2a.

This analysis indicates that the value of the executive's option declined by approximately $287,000 when the term was reduced by one year (and the volatility increased by 10 percentage points).

Performing a sensitivity analysis based on the changes in historical volatility illustrates just how sensitive the value is to changes in the volatility of the original option contract and the shortened option contract. This is presented in Exhibit 2b.

The sensitivity table presented in Exhibit 2b illustrates the decrease in the value of the option due to a combination of a reduction in its expiration period and an increase in the historical volatility assumption.

The values with a horizontal box around them are based on the historical volatility assumptions of 50 percent (the historical volatility for an option with 1.5 years remaining) and 60 percent (the historical volatility for an option with 0.5 years remaining).

Similarly, the values with a vertical box around them are based on historical volatility assumptions of 60 percent (the historical volatility of the shortened options contract) and 50 percent (the historical volatility of the original options contract).

If the historical volatility of the shortened option is unchanged from the historical volatility of the original option, the difference in the value of the options contracts would increase from approximately $287,000 to approximately $394,000. On the other hand, if the historical volatility of the original option is the same as the historical volatility of the shortened option, the difference in the value of the option contract increases from $287,000 to $466,000. In other words, as volatility increases (the historical volatility of the original option of 50 percent increases to the historical volatility of the shortened option of 60 percent), the value of the original option increases such that the difference is greater.

Based on this example, the economic damages suffered by the executive as a result of ABC's action to shorten the executive's stock options was approximately $287,000. This represents a loss of 37.5 percent of the value of the executive's current,

Exhibit 1
Stock Option Valuation & Volatility Considerations
Valuation Variable Inputs as of July 1, 2016

Stock Price $	Strike Price $	In the Money $	Number of Shares	Intrinsic Value $
40.00	38.00	2.00	100,000	200,000

Long-Term Target Price	$ 50.00	Implied Expected Growth	25%

	Shortened	Original	Change
Stock Price	$ 40.00	$ 40.00	
Risk-Free Rate	0.3%	0.4%	-0.1%
Historical Volatility	60%	50%	10%
Implied Volatility	90%	80%	10%
Implied Volatility Change	30%	30%	0%

Exhibit 2a
Stock Option Valuation & Volatility Considerations
Black Scholes-Merton Option Pricing Model—Historical Volatility

	Shortened	Original
Strike price	$ 38.00	$ 38.00
Stock price	$ 40.00	$ 40.00
Assumptions - Black-Scholes-Merton Option Pricing Model:		
Expiration date	1/1/2017	1/1/2018
Valuation date	7/1/2016	7/1/2016
Term (in years)	0.5	1.5
Risk-free rate	0.3%	0.4%
Estimated expected volatility (in %)	60%	50%
Variance (in %)	36%	25%
Call option value	$ 7.65	$ 10.52
Number of Shares	100,000	100,000
Option Value ($)	765,180	1,051,907
Change in Option Value ($)		**286,727**

shortened options. Recall that this value is based on a spread of historical volatility of 10 percentage points—that is, the shortened option historical volatility of 60 percent minus the original option historical volatility of 24 percent.

Using the sensitivity analysis presented in Exhibit 2b, we can see the effect of the same 10 percentage point spread at increasing historical volatility levels. These are presented in red in the sensitivity analysis. For example, the decrease in value between the original option and the shortened option of $359,000 is based on an original option historical volatility assumption of 60 percent and a shortened option historical volatility assumption of 70 percent (i.e., a spread of 10 percentage points). This decrease in value represents a loss of 46.9 percent of the shortened option value by reducing the remaining term of the option.

Exhibit 2c presents the results of examining the same 10 percentage point spread at increasing historical volatility levels.

As is evident in Exhibit 2c, as historical volatility levels increase, keeping the same 10 percentage point spread in volatility, the economic damage increases and becomes more material as a percentage of the option value of the shortened option contract.

Implied Volatility

Next, we examine the impact on shortening the executive's option contract using the BSM model and relying on implied volatility assumptions. Stock options on ABC shares are publicly traded.

Based on the inputs in Exhibit 1, we calculate the value of the original option and the option with the shorter remaining term using the BSM model and implied volatility assumptions. We then calculate the difference in value—that is, the damage the executive would suffer if the remaining term of the option is shortened by one year. This is presented in Exhibit 3a.

Exhibit 2b
Stock Option Valuation & Volatility Considerations
Sensitivity Analysis—Historical Volatility

Volatility - Shortened Contract	Volatility - Original Contract: 5%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
10%	2,547	79,300	258,723	443,301	627,458	809,522	988,517	1,163,701	1,334,445	1,500,202	1,660,493
20%	(93,565)	(16,811)	162,612	347,189	531,347	713,410	892,406	1,067,590	1,238,334	1,404,091	1,564,381
30%	(198,531)	(121,778)	57,646	242,223	426,380	608,444	787,440	962,623	1,133,367	1,299,124	1,459,415
40%	(305,533)	(228,779)	(49,356)	135,221	319,379	501,442	680,438	855,622	1,026,366	1,192,122	1,352,413
50%	(412,981)	(336,228)	(156,805)	27,773	211,930	393,994	572,989	748,173	918,917	1,084,674	1,244,965
60%	(520,248)	(443,495)	(264,071)	(79,494)	104,663	**286,727**	465,723	640,906	811,651	977,407	1,137,698
70%	(626,987)	(550,234)	(370,810)	(186,233)	(2,076)	179,988	358,983	534,167	704,911	870,668	1,030,959
80%	(732,963)	(656,210)	(476,787)	(292,209)	(108,052)	74,012	253,007	428,191	598,935	764,692	924,983
90%	(837,991)	(761,238)	(581,815)	(397,238)	(213,080)	(31,016)	147,979	323,163	493,907	659,664	819,955
100%	(941,915)	(865,162)	(685,738)	(501,161)	(317,004)	(134,940)	44,056	219,240	389,984	555,740	716,031

Exhibit 2c
Stock Option Valuation & Volatility Considerations
Decrease in Value at Historical Volatility Spreads

Option Historical Volatility			Decrease in Value	
Shortened	Original	Spread	$	% of Shortened Contract
60%	50%	10%	286,727	37.5%
70%	60%	10%	358,983	46.9%
80%	70%	10%	428,191	56.0%
90%	80%	10%	493,907	64.5%
100%	90%	10%	555,740	72.6%

This analysis indicates that the value of the executive's options declined by approximately $494,000 by shortening the term by one year and increasing the volatility by 30 percentage points. Based on this particular example, implied volatility is greater the closer the option is to its expiration date.

Performing a sensitivity analysis based on the changes in implied volatility illustrates just how sensitive the change in value is for changes in the implied volatility of (1) the original option contract and (2) the option with the shorter remaining term. This is presented in Exhibit 3b.

The sensitivity table presented in Exhibit 3b illustrates the increase in the value of the option by reducing the remaining term based on a range of implied volatility assumptions. As discussed in the historical volatility section, the values with a horizontal box around them are based on implied volatility assumptions from 80 percent to 90 percent. Similarly, the values with a vertical box around them are based on implied volatility assumptions from 90 percent to 80 percent.

Similar to the historical volatility discussion, we note that if the implied volatility of the option with the shorter term is unchanged from the implied volatility of the original option, the value of the difference in options contracts increases from $494,000 to $599,000.

On the other hand, if the implied volatility of the original option is the same as the implied volatility of the option with the shorter term, the value of the difference in options contracts increases even more—from $494,000 to $660,000.

We see the same trend in the implied volatility analysis as we did in the historical volatility analysis—as volatility increases (the original option implied volatility of 80 percent increases to the implied volatility of 90 percent for the option with the shorter remaining term), the value of the original option increases such that the difference between the two options is greater.

Based on this example, the economic damages suffered by the executive as a result of ABC's action to reduce the remaining term of the executive's stock options was approximately $494,000. This represents a loss of 45.6 percent of the value of the executive's current, shortened options. Recall that this value is based on a spread of implied volatility of 10 percentage points, that is, the implied volatility of 90 percent for the option with the shorter term minus the original option implied volatility of 80 percent.

Exhibit 3a
Stock Option Valuation & Volatility Considerations
Black Scholes-Merton Option Pricing Model—Implied Volatility

	Shortened	Original
Strike price	$ 38.00	$ 38.00
Stock price	$ 40.00	$ 40.00
Assumptions - Black-Scholes-Merton Option Pricing Model:		
Expiration date	1/1/2017	1/1/2018
Valuation date	7/1/2016	7/1/2016
Term (in years)	0.5	1.5
Risk-free rate	0.3%	0.4%
Estimated expected volatility (in %)	90%	80%
Variance (in %)	81%	64%
Call option value	$ 10.83	$ 15.77
Number of Shares	100,000	100,000
Option Value ($)	1,082,924	1,576,831
Change in Option Value ($)		**493,907**

Exhibit 3b
Stock Option Valuation & Volatility Considerations
Sensitivity Analysis—Implied Volatility

Volatility - Shortened Contract	Volatility - Original Contract										
	5%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
10%	2,547	79,300	258,723	443,301	627,458	809,522	988,517	1,163,701	1,334,445	1,500,202	1,660,493
20%	(93,565)	(16,811)	162,612	347,189	531,347	713,410	892,406	1,067,590	1,238,334	1,404,091	1,564,381
30%	(198,531)	(121,778)	57,646	242,223	426,380	608,444	787,440	962,623	1,133,367	1,299,124	1,459,415
40%	(305,533)	(228,779)	(49,356)	135,221	319,379	501,442	680,438	855,622	1,026,366	1,192,122	1,352,413
50%	(412,981)	(336,228)	(156,805)	27,773	211,930	393,994	572,989	748,173	918,917	1,084,674	1,244,965
60%	(520,248)	(443,495)	(264,071)	(79,494)	104,663	286,727	465,723	640,906	811,651	977,407	1,137,698
70%	(626,987)	(550,234)	(370,810)	(186,233)	(2,076)	179,988	358,983	534,167	704,911	870,668	1,030,959
80%	(732,963)	(656,210)	(476,787)	(292,209)	(108,052)	74,012	253,007	428,191	598,935	764,692	924,983
90%	(837,991)	(761,238)	(581,815)	(397,238)	(213,080)	(31,016)	147,979	323,163	**493,907**	659,664	819,955
100%	(941,915)	(865,162)	(685,738)	(501,161)	(317,004)	(134,940)	44,056	219,240	389,984	555,740	716,031

Using the sensitivity analysis presented in Exhibit 3b, we can examine a similar percentage point spread of 10 percent at increasing implied volatility levels. These are presented in red in the sensitivity analysis. For example, the $556,000 decrease in value between the original option and the option with the shorter remaining term is based on an original option implied volatility assumption of 90 percent and an implied volatility assumption of 100 percent (i.e., a spread of 10 percentage points) for the option with the shorter remaining term. This decrease in value represents a loss of 51.3 percent of the value of the executive's current, shortened options.

Exhibit 3c presents the results of examining the same 10 percentage point spread at the increased historical volatility level just discussed.

As is evident in Exhibit 3c, as implied volatility levels increase, keeping the same 10 point spread, the damage increases and becomes more material as a percentage of the option value of the original option contract.

Summary of Historical Volatility Versus Implied Volatility

This example illustrates that there is a material difference from an economic perspective (and in the case of a litigation, from an economic damages perspective) if the analyst expands the valuation analysis to a consideration of implied volatility from the conventional use of historical volatility.

Exhibit 4 presents a summary of the differences between the historical volatility analysis compared to the implied volatility analysis when examining the impact of ABC reducing the remaining term of the executive's option.

As presented in Exhibit 4, this example illustrates that a stock option analysis relying on implied volatility results in economic damages of more than 70 percent of a stock option analysis relying on historical volatility.

Recalling that if implied volatility is greater than historical volatility, the actual option trading price is higher than what it would be if the historical volatility is used in the BSM model to calculate the option trading price. Does this mean that the actual option trading price implied by the implied volatility is correct or incorrect? Put differently, does this mean that the higher implied volatility of the option is correct and the lower historical volatility of the stock price is incorrect as it relates to the time horizon to the expiration of the option? Or, is the current higher implied volatility of the option incorrect or is the lower historical volatility of the stock price correct as it relates to the time horizon to the expiration of the option?

In order to answer this question and select the correct volatility measure, the analyst should carefully examine historical and implied volatility in the context of the facts and circumstances of the company and its market outlook.

The valuation analyst should also keep in mind that implied volatility is forward-looking rather than backward-looking. Thus, the volatility implied from actual option prices includes the market's current outlook of future expectations for the value of the

Exhibit 3c
Stock Option Valuation & Volatility Considerations
Decrease in Value at Implied Volatility Spreads

Option Implied Volatility			Decrease in Value	
Shortened	Original	Spread	$	% of Shortened Contract
90%	80%	10%	493,907	45.6%
100%	90%	10%	555,740	51.3%

Exhibit 4
Stock Option Valuation & Volatility Considerations
Summary of Historical Volatility versus Implied Volatility at July 1, 2016

	Volatility		Stock Price		Change in Option Value
	Shortened Contract	Original Contract	$	Reference	$
Historical Volatility	60%	50%	40.00	Exhibit 2a	286,727
Implied Volatility	90%	80%	40.00	Exhibit 3a	493,907
Change	30%	30%			207,180
Percentage Change					**72%**

Source: As indicated above, Bloomberg, and Willamet Management Associates.

stock. However, while the price of the stock already reflects the market's future expectations, this is not reflected completely in the stock's historical volatility measure. This is because historical volatility relies on many data points over a certain period of time. One day's stock price change, though high, will not change today's six-month, daily historical volatility very much from yesterday's six-month, daily historical volatility.

Therefore, today's six-month, daily historical volatility does not fully reflect the impact of today's stock price change. Implied volatility, on the other hand, does fully reflect the impact of today's stock price change and the change in the market's outlook about future stock performance.

For this reason, the valuation analyst may generally consider implied volatility as a better measure of volatility than historical volatility. However, an examination of both is the recommended best practice before selecting a final volatility estimate.

Implied volatility may be considered a leading and concurrent indicator of volatility, while historical volatility may be considered a lagging indicator of volatility.

There are, however, other perspectives that the valuation analyst should consider. These perspectives—time horizon and valuation changes—are discussed next.

Time Horizon

The time horizon selected is an important factor in estimating volatility. As mentioned above, different perspectives that the analyst may adopt may often result in widely disparate volatility conclusions.

Historical volatility may be measured in frequencies on a daily, weekly, and even monthly basis. In addition, historical volatility is measured by examining the stock prices over any time period.

For example, the analyst can examine historical volatility by analyzing daily prices during the prior (1) one month period, (2) three month period, (3) one year period, (4) multi-year period, or (5) any other unique period of time. Similarly, historical volatility may be examined by analyzing the price at the end of each week during the same time periods. In some instances, only month-end prices are analyzed.

As the duration between observations increases, fewer data points will be available to calculate the volatility. This may result in a lower level of reliability on this data. The analyst must weigh the potential benefits of selecting longer observation frequencies to calculate volatility against the effect that this might have on reliability due to the fewer data points.

Using the historical volatility measure is limited in instances where there are not enough data points to provide a statistically meaningful measure of volatility. This problem does not exist when considering implied volatility. As such, relying on implied volatility in those instances may provide a better and more reliable measure of volatility.

Implied volatility is measured by examining stock options that will expire in the future. Unlike historical volatility time horizons, which offer much greater flexibility in selecting the time horizon, implied volatility time horizons are limited to (1) the number of stock options issued and outstanding at a particular time and (2) the expirations of those options.

Stock option prices for a specific expiration date are different based on their strike price because their intrinsic values are different. Theoretically, however, the implied volatility of options expiring on the same date should be the same regardless of their strike prices. In other words, while their actual option prices are not the same, because of their different strike prices, their implied volatility should, theoretically, be the same. If they are not the same, this may add an additional layer of complexity in selecting the appropriate options at a given expiration date.

Stock options that have the same expiration date but have different implied volatilities should be analyzed to determine whether the magnitude of the differences is material. Then, the valuation analyst may select the implied volatility of one particular stock option at a particular strike price or some or all of the stock options expiring on that same day.

Changes in Volatility

Volatility changes over time. For any one specific procedure that the analyst applies to examine volatility at a particular date, when that same procedure is applied at a different date, it may yield a different volatility conclusion. This is true for historical volatility as well as implied volatility.

If the volatility, whether measured historically or implied, does not change over different dates, this may give support for a particular current volatility estimate.

For example, assuming implied volatility today for a three-month option is 50 percent, if the analyst examines the same three-month option at various intervals in the past (three months ago, six months ago, nine months ago, one year ago) and finds the same 50 percent volatility, this fact pattern lends considerable support for today's 50 percent volatility indication.

The analyst may consider changes in volatility over time in performing these procedures to estimate volatility. Depending on the purpose of the valuation and the scope of the assignment, the change in volatility over time may call for further analysis before reaching the conclusion of the option value.

Continuing with our example, let's look to see how things change for the same company at a different time horizon and different volatilities. In this part of the example, the executive of ABC holds the same options at the same strike price and at the same expiration, but in this example the valuation date is January 1, 2016, and the expiration is now 2.0 years. If ABC reduces the executive's option expiration period by one year, how would the value of his options change and would the shorter expiration period in this instance have damaged him economically?

The details of the stock option and the BSM model inputs for this scenario are presented in Exhibit 5.

We will perform the same analysis as in the previous discussion for the new earlier valuation date of January 1, 2016.

Historical Volatility

Based on the inputs in Exhibit 5, we calculate the value of the original option and the option with the shorter remaining term using the BSM model and historical volatility assumptions. We then calculate the difference in value, that is, the value that the executive would be giving up if the term of the option is shortened by one year. This is presented in Exhibit 6a.

This analysis indicates that the value of the executive's option declined by approximately $48,000 by shortening the term by one year and leaving the volatility relatively unchanged.

Performing a sensitivity analysis based on the changes in historical volatility illustrates just how sensitive the change in value is for changes in the historical volatility of the original option contract and the shortened option contract. This is presented in Exhibit 6b.

The sensitivity table presented in Exhibit 6b illustrates the decrease in the value of the option due to a combination of a reduction in its expiration period and an increase in the historical volatility assumption. Recall that we noted in our earlier discussion that if the historical volatility of the option with the shorter remaining term is unchanged from the historical volatility of the original option, the difference in the value of the options contracts would increase in this particular case. Conversely, if the historical volatility of the original option increases, the difference in the value of the options contracts also increases.

Based on this example, the economic damages suffered by the executive as a result of ABC's action to reduce the remaining term of the executive's stock option was approximately $48,000. This represents a loss of 4.8 percent of the value of the executive's current, shortened option. In this scenario, this value is based on a 10 percentage point spread in historical volatility.

Using the sensitivity analysis presented in Exhibit 6b, we can examine the same 10 percentage point spread at increasing historical volatility levels. These are presented in red in the sensitivity analysis.

Exhibit 5
Stock Option Valuation & Volatility Considerations
Valuation Variable Inputs as of January 1, 2016

Stock Price $	Strike Price $	In the Money $	Number of Shares	Intrinsic Value $
44.00	38.00	6.00	100,000	600,000

Long-Term Target Price	$ 49.00	Implied Expected Growth	11.4%

	1/1/2017	1/1/2018	Change
Stock Price	$ 44.00	$ 44.00	
Risk-Free Rate	0.4%	0.5%	-0.1%
Historical Volatility	40%	30%	10%
Implied Volatility	60%	50%	10%
Implied Volatility Change	20%	20%	0%

Exhibit 6a
Stock Option Valuation & Volatility Considerations
Black Scholes-Merton Option Pricing Model—Historical Volatility

	Shortened		Original
Strike price	$ 38.00		$ 38.00
Stock price	$ 44.00		$ 44.00
Assumptions - Black-Scholes-Merton Option Pricing Model:			
Expiration date	1/1/2017		1/1/2018
Valuation date	1/1/2016		1/1/2016
Term (in years)	1		2
Risk-free rate	0.4%		0.5%
Estimated expected volatility (in %)	40%		30%
Variance (in %)	16%		9%
Call option value	$ 10.01		$ 10.49
Number of Shares	100,000		100,000
Option Value ($)	1,001,020		1,049,258
Change in Option Value ($)			**48,238**

Exhibit 6b
Stock Option Valuation & Volatility Considerations
Sensitivity Analysis—Historical Volatility

		Volatility - Original Contract										
		5%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Volatility - Shortened Contract	10%	12,069	49,294	218,397	422,346	633,734	845,209	1,053,586	1,256,998	1,454,142	1,644,030	1,825,885
	20%	(82,330)	(45,105)	123,998	327,947	539,335	750,811	959,188	1,162,599	1,359,743	1,549,631	1,731,487
	30%	(216,164)	(178,939)	(9,836)	194,114	405,501	616,977	825,354	1,028,765	1,225,909	1,415,797	1,597,653
	40%	(362,040)	(324,815)	(155,711)	**48,238**	259,625	471,101	679,478	882,889	1,080,033	1,269,921	1,451,777
	50%	(512,014)	(474,788)	(305,685)	(101,736)	109,652	321,127	529,504	732,916	930,060	1,119,948	1,301,803
	60%	(662,947)	(625,722)	(456,619)	(252,670)	(41,282)	170,193	378,570	581,982	779,126	969,014	1,150,869
	70%	(813,248)	(776,023)	(606,920)	(402,971)	(191,583)	19,892	228,269	431,681	628,825	818,713	1,000,569
	80%	(961,942)	(924,717)	(755,614)	(551,665)	(340,277)	(128,802)	79,576	282,987	480,131	670,019	851,875
	90%	(1,108,349)	(1,071,124)	(902,021)	(698,072)	(486,684)	(275,209)	(66,832)	136,580	333,724	523,612	705,467
	100%	(1,251,952)	(1,214,726)	(1,045,623)	(841,674)	(630,286)	(418,811)	(210,434)	(7,022)	190,122	380,010	561,865

For example, the decrease in value between the original option and the option with the shorter remaining term of $110,000 is based on an original option historical volatility assumption and a shorter-term option historical volatility assumption of 10 percentage points. This decrease in value represents a loss of 11.0 percent of the value of the executive's current, shortened options.

Exhibit 6c presents the results of examining the same 10 percentage point spread at increasing historical volatility levels.

As is evident in Exhibit 6c, as historical volatility levels increase, keeping the same 10 percentage point spread, the decrease in value (from the original option to the shortened option) increases and becomes more material as a percentage of the option value of the shortened option contract.

Implied Volatility

Based on the inputs in Exhibit 5, we calculate the value of the original options and the shorter-term options using the BSM model and implied volatility assumptions. We then calculate the difference in value, that is, the value the executive would be giving up if the remaining term of the option is shortened by one year. This is presented in Exhibit 7a.

Exhibit 6c
Stock Option Valuation & Volatility Considerations
Decrease in Value at Historical Volatility Spreads

Option Historical Volatility			Decrease in Value	
Shortened	Original	Spread	$	% of Shortened Contract
40%	30%	10%	48,238	4.8%
50%	40%	10%	109,652	11.0%
60%	50%	10%	170,193	17.0%
70%	60%	10%	228,269	22.8%
80%	70%	10%	282,987	28.3%
90%	80%	10%	333,724	33.3%
100%	90%	10%	380,010	38.0%

This analysis indicates that the value of the executive's options declined by approximately $170,000 by shortening the term by one year and increasing the volatility by approximately 10 percentage points.

Performing a sensitivity analysis based on the changes in implied volatility illustrates just how sensitive the change in value is for changes in the implied volatility of the original option and the shorter-term option. This is presented in Exhibit 7b.

The sensitivity table presented in Exhibit 7b illustrates the increase in the value of the option by reducing the term of its expiration based on a range of implied volatility assumptions. As discussed in the historical volatility section, the values with a horizontal box around them are based on implied volatility assumptions from 50 percent to 60 percent. Similarly, the values with a vertical box around them are based on implied volatility assumptions from 60 percent to 50 percent.

Similar to the earlier discussions of sensitivity analyses, as illustrated in Exhibit 7b, if the implied volatility of the shorter-term option is unchanged from the implied volatility of the original option, the value of the difference in options contracts increases materially from $170,000 to $322,000.

Moreover, if the implied volatility of the original option is the same as the implied volatility of the shorter-term option, the difference in the value of the options increases even more—from $170,000 to $379,000.

Based on this example, the economic damages the executive suffered as a result of ABC's action to reduce the term of the executive's stock options was approximately $170,000. This represents a loss of 13.1 percent of the value of the executive's current options with a shorter expiration period.

Exhibit 7a
Stock Option Valuation & Volatility Considerations
Black Scholes-Merton Option Pricing Model—Implied Volatility

	Shortened		Original
Strike price	$ 38.00		$ 38.00
Stock price	$ 44.00		$ 44.00
Assumptions - Black-Scholes-Merton Option Pricing Model:			
Expiration date	1/1/2017		1/1/2018
Valuation date	1/1/2016		1/1/2016
Term (in years)	1		2
Risk-free rate	0.4%		0.5%
Estimated expected volatility (in %)	60%		50%
Variance (in %)	36%		25%
Call option value	$ 13.03		$ 14.74
Number of Shares	100,000		100,000
Option Value ($)	1,303,186		1,473,606
Total Change in Option Value ($)			**170,420**

Exhibit 7b
Stock Option Valuation & Volatility Considerations
Sensitivity Analysis—Implied Volatility

Volatility - Shortened Contract	Volatility - Original Contract										
	5%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
10%	12,072	49,439	218,887	423,154	634,836	846,584	1,055,211	1,258,846	1,456,186	1,646,242	1,828,234
20%	(82,581)	(45,214)	124,234	328,500	540,183	751,931	960,557	1,164,193	1,361,533	1,551,588	1,733,581
30%	(216,659)	(179,293)	(9,845)	194,422	406,104	617,852	826,479	1,030,114	1,227,455	1,417,510	1,599,503
40%	(362,764)	(325,397)	(155,949)	48,318	260,000	471,748	680,375	884,010	1,081,350	1,271,406	1,453,398
50%	(512,953)	(475,587)	(306,139)	(101,872)	109,810	321,558	530,185	733,820	931,161	1,121,216	1,303,209
60%	(664,092)	(626,725)	(457,277)	(253,011)	(41,329)	**170,420**	379,046	582,681	780,022	970,077	1,152,070
70%	(814,587)	(777,220)	(607,772)	(403,505)	(191,823)	19,925	228,552	432,187	629,527	819,582	1,001,575
80%	(963,462)	(926,096)	(756,648)	(552,381)	(340,699)	(128,951)	79,676	283,311	480,652	670,707	852,700
90%	(1,110,039)	(1,072,672)	(903,224)	(698,958)	(487,275)	(275,527)	(66,901)	136,734	334,075	524,130	706,123
100%	(1,253,797)	(1,216,430)	(1,046,982)	(842,716)	(631,034)	(419,285)	(210,659)	(7,024)	190,317	380,372	562,365

Exhibit 7c
Stock Option Valuation & Volatility Considerations
Decrease in Value at Implied Volatility Spreads

Option Implied Volatility			Decrease in Value	
				% of Shortened
Shortened	Original	Spread	$	Contract
60%	50%	10%	170,420	13.1%
70%	60%	10%	228,552	17.5%
80%	70%	10%	283,311	21.7%
90%	80%	10%	334,075	25.6%
100%	90%	10%	380,372	29.2%

Recall that this value is based on a spread of historical volatility of 10 percentage points—that is, the shorter-term option implied volatility of 60 percent minus the original option implied volatility of 50 percent.

Using the sensitivity analysis presented in Exhibit 7b, we can examine a similar 10 percentage point spread in implied volatility levels. These are presented in red in the sensitivity analysis.

For example, the decrease in value between the original option and the shorter-term option of $229,000 is based on an original option implied volatility assumption of 70 percent and a shorter-term option historical volatility assumption of 60 percent (i.e., a spread of 10 percentage points). This decrease in value represents a loss of 17.5 percent of the value of the executive's original options.

Exhibit 7c presents the results of examining the same 10 point spread in implied volatility levels.

As is evident in Exhibit 7c, as implied volatility levels increase, keeping the same 10 point spread, the damage increases and becomes more material as a percentage of the current, shortened option value.

Summary of Historical Volatility Versus Implied Volatility

Similar to the previous example as of July 1, 2016, this example also illustrates that, as of January 1, 2016, there is a material difference from an economic perspective (and in the case of a litigation, from an economic damages perspective) if the analyst expands the valuation analysis beyond the conventional use of historical volatility to considering implied volatility.

Exhibit 8 presents a summary of the differences between the historical volatility analysis compared to the implied volatility analysis when examining the impact of ABC reducing the remaining term of the executive's option.

As presented in Exhibit 8, this example illustrates that a stock option analysis using implied volatility results in economic damages of greater than two and a half times that of a stock option analysis using historical volatility.

This analysis as of January 1, 2016, allowed us to examine the same option analysis of ABC stock as of an earlier point in time, according to the second perspective related to time horizon. Since the historical volatility and implied volatility were different as of the January 1, 2016, analysis, this also allows us to examine the same option analysis of ABC stock having differing volatilities.

As presented in Exhibit 9, the shorter time horizon resulted in more than twice the economic damages by relying on implied volatility. A similar analysis can be performed for historical volatility. The valuation analyst should assess which volatility measure is the most reliable and supportable.

This example illustrates the three perspectives in estimating volatility:

1. Implied volatility
2. Time horizon
3. Change in volatility over time

All of these perspectives affect the valuation of stock options.

Exhibit 8
Stock Option Valuation & Volatility Considerations
Summary of Historical Volatility versus Implied Volatility
at January 1, 2016

	Volatility		Stock		Change in
	Shortened	Original	Price		Option Value
	Contract	Contract	$	Reference	$
Historical Volatility	40%	30%	44.00	Exhibit 6a	48,238
Implied Volatility	60%	50%	44.00	Exhibit 7a	170,420
Change	20%	20%			122,182
Percentage Change					**253%**

Source: As indicated above, Bloomberg, and Willamet Management Associates.

Other Considerations

Another financial metric that the valuation analyst may consider in determining the proper volatility assumption when valuing stock

options is third-party analysts' consensus estimates of the long-term target stock price for a particular company. Since this is a forward-looking metric, it only applies to implied volatility and not to historical volatility.

Our example above illustrates that this metric does change over different points in time, as presented in Exhibits 1 and 5 above. While the long-term target price estimate is not an input in the BSM model, it is a data point that is considered by both the investment community and the valuation community in assessing the value of stock options vis-à-vis its current stock price.

Exhibit 9
Stock Option Valuation & Volatility Considerations
Impact of Time Horizon on Option Value Changes Using Implied Volatility

Volatility	Valuation Date	Time Horizon Shortened	Time Horizon Original	Reference	Change in Option Value $
Implied	July 1, 2016	0.5	1.5	Exhibit 3a	493,907
Implied	January 1, 2016	1.0	2.0	Exhibit 7a	170,420
Increase in Economic Damages					323,488
Percentage Change					**190%**

Source: As indicated above, Bloomberg, and Willamet Management Associates.

This long-term target price estimate metric is also important in assessing the impact of a company reducing the remaining term of an executive's stock option. As the long-term target stock price increases, a shorter-term stock option would preclude the executive from enjoying the increase in the anticipated intrinsic value and the increase in the actual option value of the stock option.

Understanding the long-term target stock price estimate also aids the executive in deciding what action to take in exercising the stock option at the date the shorter term option expires.

When exercising stock options, the executive's stock options in our example, and the holder of a stock option in general, has three alternatives:

1. Take a full position in the stock—exercise the option and hold the stock
2. Take a partial position in the stock—exercise the option and sell enough stock to cover the tax liability related to exercising the option
3. Cash out (take a zero position in the stock)—exercise the option and sell all the stock

Taking a Full Position in the Stock of a Stock Option

Taking a full position in the underlying stock of a stock option is the first alternative the holder of a stock option may take at or prior to the expiration date. This involves exercising the option, paying the strike price of the option for the shares of stock, and paying the taxes on the intrinsic value (the extent to which the option is in-the-money) of the option at the exercise date.

The holder of a stock option would select this alternative if the executive believes the stock price will increase in the future, thereby enjoying further profits in excess of the intrinsic value achieved through exercising the option.

However, even if the holder of the option believes the stock price will increase in the future, the executive may not be able to select this alternative. This is because the option holder may not have enough money to pay the strike price and the taxes. This might be the case especially with very large positions in stock options.

In the case of our example with the executive of ABC, the cash required to exercise and take a position in the stock is $3.8 million ($38 strike price * 100,000 shares). At a stock price of $42, the tax liability of exercising the option at a 40 percent tax rate is $160,000 ([$42 stock price minus $38 strike price] * 100,000 shares * 40 percent). Therefore, the executive would have to pay a total of $3.96 million in order to take a full position in the stock of ABC.

In this case (taking a full position in the stock upon exercising ABC stock options), the executive would have zero economic damages of exercising early at the shorter-term expiration date. This is because, upon exercise, the executive now owns the underlying stock and would, therefore, enjoy any increase in the stock price. In other words, exercising early and taking a full position versus exercising later and taking a full position would not change the executive's economic position compared to the original expiration date.

Taking a Partial Position in the Stock of a Stock Option

Taking a partial position in the underlying stock of a stock option is the second alternative available to

the holder of a stock option at or prior to the expiration date. There are many partial positions a stock option holder may take.

Let's consider the alternative where the stock option holder wishes to take the largest position the executive can in the stock while not paying any money—that is, undergoing a cashless exercise of the stock option. This alternative is a favorite among many stock option holders for the following reasons:

1. It does not require any cash outlay to exercise the option.
2. It does not provide any cash to the stock option holder upon exercise.
3. Most importantly, it allows the stock option holder to hold the maximum number of shares of the stock as possible, given the first two constraints.

In effect, this alternative is truly a cashless exercise. That is, the stock option holder exercising the stock option neither receives cash nor pays cash.

Executing this alternative involves exercising the stock option and then selling only those number of shares required to pay for (1) exercising the stock option (the strike price multiplied by the number of shares) and (2) the taxes (current stock price minus strike price multiplied by the tax rate).

In the case of our example with the ABC executive, if the stock price increases over the next six months, the economic damages would be limited to the lost value for the number of shares the executive sold in order to satisfy the cash requirements (both for acquiring the stock at the strike price and paying the tax liability for the intrinsic value) for exercising the stock options. There would be no damages associated with the stock the executive received because the executive would enjoy any stock price increase over the subsequent six months just like the original option.

In general, the economic damages related to the lost value for the number of shares the executive sold in order to satisfy the cash requirements for exercising the stock options would typically only be considered "damages" if the executive did not have any cash at the shortened expiration date in order to exercise the option and take a full position in the stock and the executive was, therefore, forced to sell the stock.

If the executive did have the cash to exercise the option and take a full position in the stock, an argument may be made that there would not be any economic damages suffered by the executive related to being forced to accept the shorter-term option.

Cashing Out—Taking a Zero Position in the Stock of a Stock Option

Finally, cashing out by taking a zero position in the underlying stock is the third alternative available to the holder of a stock option at or prior to the expiration date.

In this alternative, the option holder simply exercises the option, sells all the underlying stock, pays for both acquiring the stock at the strike price and paying the tax liability for the intrinsic value, and receives the remaining cash proceeds.

In the case of the ABC executive, since the stock option holder has exercised the stock options and sold all of the underlying stock, the economic damages, if the stock price increases over the subsequent six months from the shortened expiration date, is the maximum among the three alternatives discussed here.

However, to the extent that the second alternative is easily available to the executive (i.e., performing a cashless exercise and taking a position in the stock), then there may be a duty to mitigate his damages. Therefore, the economic damages that may be available to the executive may be limited to what is included under the second alternative.

Summary and Conclusion

We discussed several perspectives that a stock option analyst should consider when estimating volatility. Estimating volatility involves an examination of (1) historical volatility versus implied volatility, (2) the appropriate time horizon over which volatility should be measured, and (3) the changes in volatility over different periods of time in the past.

Performing a comprehensive analysis of volatility is important because (1) the valuation of stock options may be very sensitive to even small changes in volatility and (2) different perspectives to estimate volatility may result in materially different conclusions.

The following points summarize important insights regarding the above discussion of stock option volatility considerations:

1. As volatility increases, the value of a stock option, holding all other variables constant, also increases.
2. Historical volatility may be measured in multiple frequencies and over multiple time periods. Each frequency and time period used to measure volatility may result in disparate volatility indications. Therefore, care must be exercised in considering these multiple

methods of measuring historical volatility and the selected method.

3. Implied volatility time periods are more limited than historical volatility.
4. Implied volatility for stock options of a particular company expiring at a specific expiration date should, theoretically, be the same. If they are not, the analyst should exercise care in investigating the reasons for the differences, the magnitude of the differences, and selecting the appropriate implied volatility measure.
5. Examining volatility, both historical and implied, using the same parameters at different points in the past is an important perspective in testing the valuation analyst's selected volatility. If the volatility measures at different points in the past are consistent with the current volatility measure, this may provide reasonable support for the selected volatility.
6. In the ABC executive's stock option example, (a) shortening the expiration of a stock option increases the volatility and decreases the stock option value and (b) doing so at a valuation date that is closer to the expiration date increase the volatility more and decreases the value more.
7. Implied volatility is forward-looking and therefore captures the future expectations of the market and the company. Because of this feature of implied volatility, which historical volatility does not have, the valuation analyst may consider implied volatility preferred to historical. At a minimum, the analyst should consider implied volatility along with historical volatility in selecting a volatility measure for valuing a stock option.
8. Another benefit of implied volatility over historical volatility is that the speed and the magnitude of a company's current and expected performance may be reflected in the volatility measure. Implied volatility is affected by company news daily. On the other hand, even a large change in the stock price of the company will not result is a commensurate change in the company's historical volatility. Consequently, the analyst may generally consider implied volatility as a better measure of volatility than historical volatility. However, an examination of both is the recommended best practice before selecting a final volatility estimate.
9. The analysts' consensus estimates of the long-term target stock price for a particular company provides meaningful information about the company's outlook. This financial metric is only captured in implied volatility.
10. Implied volatility may be considered a leading and concurrent indicator of volatility, while historical volatility may be considered a lagging indicator of volatility.
11. Estimating the volatility measure is an important input, if not the most important input, in performing stock option valuations, or, for that matter, any other type of valuation that requires a volatility input.

As described above, estimating volatility can be very complex. An analysis of volatility should (1) be comprehensive, (2) include an examination of the three perspectives discussed in this article, (3) consider the important understandings and insights discussed in this article, (4) be conducted in the context of the facts and circumstances of a particular company and the market in which it competes, (5) consider the characteristics and requirements of the specific purpose of the valuation, and (6) be reasonable, explainable, and supportable.

Notes:

1. John C. Hull, *Fundamentals of Futures and Options Markets*, 9th ed. (New York: Pearson, 2016), 308.
2. Ibid.
3. This means that when the historical volatility of the shortened option remains the same as (unchanged from) the historical volatility of the original option of 50 percent, how does the change in the option values between the shortened option and the original option behave? Is it greater, unchanged, or less? This same analysis is repeated in subsequent scenarios in this article.
4. The assumption implicit in this article is that the stock option is an American-style option in which the vested stock option may be exercised any time before or at the date of expiration. European-style options may only be exercised on the expiration date of the option. Because of the flexibility in the exercise date, the value of an American-style option is usually greater than the value of European-style options.

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