Cost of Capital Theory and Application for Fair Value Controversy Matters

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In practice, applying a size premium to estimate the cost of equity capital as part of a business valuation engagement is a generally accepted analytical procedure. Before selecting and applying a size premium, however, the analyst should consider all of the potential issues related to incorporating a size premium in the cost of capital estimation.

Introduction

Valuation analysts ("analysts") often use the income approach to value a closely held business enterprise. An important financial variable of the business valuation income approach involves the selection of the present value discount rate.

The cost of equity capital is an important component of the present value discount rate. Common equity capital estimation models used in the closely held business valuation process include the build-up rate model and the modified capital asset pricing model ("MCAPM").[1]

As a component of these generally accepted models, analysts often include a size premium alpha adjustment factor as part of the cost of equity estimation procedure.

This discussion considers the following topics:

1. Empirical evidence supporting the size premium adjustment
2. Observations regarding the size premium
3. Observations regarding the CRSP size premium 10th decile category
4. Liquidity issues that may account for the size premium
5. Certain Delaware Chancery Court decisions involving a size premium discussion

The common formula for the build-up model ("BUM") to estimate the cost of equity capital is presented as follows:

$$K_e = R_f + E_{RP} + I_{RP} + S_{RP} + \alpha$$

where:

K_e = Cost of equity capital
R_f = Risk-free rate of return
E_{RP} = Long-term equity risk premium
I_{RP} = Industry-related equity risk premium
S_{RP} = Size-related equity risk premium
α = Unsystematic equity risk premium

There is general consensus among analysts as to the appropriate risk-free rate of return to use in the BUM. Analysts commonly select the market yield on the 20-year U.S. Treasury bond as the risk-free rate of return component. If investment duration is less than 20 years, an analyst may select a risk-free rate of return with an investment duration commensurate with the specific investment duration.

The selected long-term equity risk premium ("ERP") is not as consistently applied among analysts. Certain analysts advocate the use of a more normalized equity risk premium, of say 5 percent. Other analysts elect to use the variables included in the CRSP Decile Size Premium Study published in the *2017 Valuation Handbook-U.S. Guide to Cost of Capital* ("*Valuation Handbook*") in Appendix 3.[2]

The *Valuation Handbook* ERP data is the most commonly cited, providing an estimated ERP premium of around 6 percent.

Other components of the BUM cost of equity estimate often include an industry-related equity risk premium, a size-related equity risk premium, and an unsystematic equity risk premium. By adding an industry-related risk premium, general industry risk is incorporated in the cost of equity.

This general industry risk premium is not specifically addressed in the long-term equity risk premium component. The industry risk component of the build-up cost of equity capital incorporates systematic risk, in much the same way that beta incorporates industry risk in the CAPM.

The next two components of the BUM are the size-related equity risk premium and the unsystematic equity risk premium. An overview of the size-related equity risk premium is presented in the following pages of this discussion.

The unsystematic equity risk premium component is often applied by analysts. This component is used to incorporate risk that is specific to the subject investment—that is, lack of management talent, potential labor issues specific to the subject company, potential of losing a key client or key personnel, and/or potential cost/risk not identified in financial projections, and so forth.[3]

The basic CAPM formula for estimating the cost of equity capital for publicly traded security analysis is presented as follows:[4]

$$Ke = Rf + [\beta \times E_{RP}]$$

where:

K_e = Cost of equity capital
R_f = Risk-free rate of return
β = Industry beta
E_{RP} = Long-term equity risk premium

Analysts use many of the same components for the CAPM formula that are used in the BUM. That is, it is common for analysts to rely on the same risk-free rate of return and long-term equity risk premium component factors when presenting both the BUM and the CAPM to estimate the cost of equity. The one distinguishing CAPM factor is beta.[5]

Beta, in general terms, is used to incorporate market risk (general equity risk and industry risk) in an equity cost of capital estimate.

Further adjustments to CAPM include (1) the size-related equity risk premium component and (2) the unsystematic equity risk premium component. By making these alpha adjustments, the CAPM becomes the modified CAPM (or "MCAPM").

The MCAPM formula for estimating the cost of equity capital for use in a closely held business valuation is presented as follows:

$$K_e = R_f + [\beta \times E_{RP}] + S_{RP} + \alpha$$

where:

K_e = Cost of equity capital
R_f = Risk-free rate of return
β = Industry beta
E_{RP} = Long-term equity risk premium
S_{RP} = Size-related equity risk premium
α = Unsystematic equity risk premium

The MCAPM and BUM provide generally consistent and easy to replicate cost of equity capital calculations.

Empirical Evidence Supporting the Size Premium

It is generally accepted that, based on empirical observation, small companies are a greater investment risk than larger companies and, therefore, smaller companies have greater cost of capital than larger companies. In other words, there is a significant (negative) relationship between size and historical equity returns.

It is also generally accepted that small companies have certain risk characteristics that are more prevalent than in larger companies.

These small company risk characteristics include the following:

1. Potential competition issues (it is easier to enter the market and compete with small companies, while larger companies have resources to mitigate competitive challenges)
2. Economic issues and concern (larger companies can better cope with economic downturn than small companies)
3. Limited access to capital (small companies can find it difficult to obtain funding while larger companies typically have more options for funding)
4. Management depth concerns (large companies do not have key employee concerns in the same way that smaller companies do)
5. Customer concentration and product concentration risk (small companies are typically not as diversified in product offerings and are often beholden to a small group of customers)
6. Liquidity concerns and lack of market coverage (small companies do not enjoy the same level of analyst coverage and small

company stock is typically less liquid than larger companies)

Rolf Banz, in a 1981 study, is credited and commonly cited for his research focusing on the empirical relationship between equity return and the total market value of NYSE common stocks.

According to Banz, smaller firms have higher risk-adjusted returns, on average, than larger firms. For the approximate 40 years covered in the study, on average, small firms recorded larger risk-adjusted returns than large firms traded on the NYSE. The Banz study found that the size effect did not exhibit linear attributes; however, the size effect was found to be more pronounced in smaller firms.

Another noteworthy finding in the Banz study was that the study suggests no theoretical foundation for the size effect. It concluded no determination as to whether the size effect factor is due to size itself or whether size is just a proxy for one or more true but unknown factors correlated with size. According to Banz, the size effect exists but it is not clear why it exists.

The *Valuation Handbook* is a common source reference for the size premium risk adjustment. The *Valuation Handbook* provides empirical evidence of the size premium phenomena. It is published as an annual reference book, along with three quarterly updates.

The *Valuation Handbook* defines the size premium as the difference between actual historical excess returns and the excess return predicted by beta (referred to as the "CRSP size premium").[8]

Exhibit 1 presents empirical evidence of the CRSP size premium, as published in the most recent *Valuation Handbook*.[9]

As presented in Exhibit 1, the empirical data illustrates stock market returns by size decile for the 1926 to 2016 time period.[10]

The annual stock market returns are separated into 10 deciles based on market capitalization. As the deciles get smaller, from 1 to 10, the historical stock market returns increase. The standard deviation of stock return portfolios also increases as deciles get smaller.[11] This increase in the standard deviation reflects noise in the data.

A review of Exhibit 1 indicates that the most statistical data noise in the 10 decile stratification is in the 10th decile classification.

Other empirical evidence, in support of the small capitalization size premium adjustment, is provided by international equity market data. For example, in the United Kingdom, a study conducted using its equity markets concluded a small capitalization stock premium of around 7 percent.[12] The U.K. study was conducted using equity market data from 1955 to 1984.

In 2015, an equity risk premium analysis study of small capitalization stocks in 23 global markets was conducted by Dimson, March, and Staunton.[13]

In the 23 global equity markets small cap stocks outperformed in every market except for Norway, Finland, and the Netherlands. In general, evidence of the small capitalization stock premium is more prevalent in developed markets than in emerging markets.

Observations Regarding the Size Premium

There are several observations regarding the data used to calculate the size premium adjustment. A few of these observations include the following:

- The small capitalization premium has disappeared in recent years.
- A premium is unduly influenced by stocks with less than $5 million in market capitalization.
- The supporting data are too noisy to calculate a meaningful size premium estimate due to the evidence of significant standard errors and seasonality.

Exhibit 1
Current 10 Decile Statistics
As of December 31, 2016

Decile	Market Capitalization (in $ millions)	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
1 - Largest	24,361.7 to 609,163.5	9.31	11.05	18.92
2	10,784.1 to 24, 233.7	10.56	12.82	21.49
3	5,684.0 to 10,711.2	11.04	13.57	23.35
4	3,520.6 to 5,676.7	10.85	13.80	25.56
5	2,393.7 to 3,512.9	11.49	14.62	26.18
6	1,571.2 to 2,390.9	11.37	14.81	27.11
7	1,033.3 to 1,570.0	11.58	15.41	29.02
8	569.3 to 1,030.4	11.56	16.14	33.01
9	263.7 to 567.8	11.56	16.97	37.18
10 - Smallest	2.5 to 263.0	13.31	20.27	42.45

Source: *2017 Valuation Handbook: U.S. Guide to Cost of Capital* , Exhibit 4-1 and Appendix 3.

- There may be other factors than size that contribute to greater small capitalization stock returns compared to large capitalization stock returns, such as:
  - bid/ask spread bias,
  - delisting bias,
  - transaction costs, and
  - liquidity.

It is generally accepted that the small capitalization stock premium was observable prior to 1980. However, it appears that the small capitalization stock premium has decreased since 1981.[14]

The Horwitz study found that during the period of 1963 to 1981, the annualized return difference between small and large firms was greater than 13 percent.[15]

However, the study also found that, during the period of 1981 to 1997. the annualized difference was negative 2 percent.[16]

Perhaps the reason for the small capitalization stock premium decrease is twofold:

1. Market corrections induced by investor understanding of the small capitalization premium phenomena
2. External economic and technological changes in the way the securities are bought and sold

As suggested in the Horowitz study, a trend toward passive investing using index funds that give more weight to large capitalization stocks may be a reason for increases in capital gain performance of large capitalization stocks.[17]

Because small capitalization stock performance as compared to large capitalization stock performance over short-term duration is typically more erratic, measurement over a longer term is preferred. For holding measurement periods of 1 year, 5 years, 10 years, 20 years, and 30 years, small capitalization stocks outperform large capitalization stocks a majority of the time—measured from 1926 to 2016.[18]

As the measurement periods increase, so does the likelihood of small capitalization stock outperformance of large capitalizations stocks.

Small capitalization stock performance is cyclical, and cyclicality should be expected. Small capitalization stock returns are variable and somewhat volatile. According to one analyst, if small companies always earned more than large companies, then small companies would not be a riskier investment endeavor in the aggregate.[19]

It is also pointed out that bond prices occasionally outperform equities. In 2014, long-term U.S. government bonds outperformed the S&P 500 Index by 10 percent.[20]

Even over a long period of time, which provides the strongest support for the existence of a small cap premium, the Horowitz study found that removing stocks with less than $5 million in market capitalization causes the small firm effect to vanish.[21]

According to the Horowitz study, the percentage of companies with stock prices of less than $2 per share was greater in the period of 1982 to 1997 than in the period of 1963 to 1981.[22]

In the smallest decile, 11.7 percent of companies traded at prices less than $2 a share between 1963 to 1981. In the 1982 to 1997, the percentage of companies traded at prices less than $2 per share in the smallest decile was 29.7 percent.

In general, historical equity returns exhibit unpredictable variability. Estimates of security risk using historical equity returns reflect noise in the form of large standard errors.[23]

As presented in Exhibit 1, as decile classifications of stock increase—correlated with smaller capitalization stocks—the standard deviation increases. The standard errors by decile class suggest that the small capitalization premium is fragile—almost to the point of lacking statistical significance.[24]

The January effect, seasonality of small capitalization stock returns, is a well-documented phenomenon. The January effect is described as the empirical observation that rates of return for small stocks have, on average, performed better in January than in other months of the year.[25]

In the Horowitz study, the average monthly return in the month of January for small capitalization stocks was 10.20 percent as compared to 0.73 for the average monthly return for February to December.[26]

The Horowitz study calculated the premium using NYSE, AMEX (now NYSE MKT), and Nasdaq stock returns for the period of 1963 to 1997. Other studies have reached similar conclusions. Although the January effect is interesting, it does not disprove that a size premium exists.

It is an unsettled discussion point that the bid/ask spread adds a certain bias to stock returns.[27] This observation is primarily focused on less liquid companies that have larger bid/ask spreads. Most of the small-size effect studies (such as the SBBI equity study prepared by Morningstar and the CRSP equity study prepared by Duff & Phelps) use the CRSP database, which relies on the closing stock price to measure rates of return.

For thinly traded stocks, the ask price is not always a realistic price. Because the small-size effect studies measure size using portfolio returns calculated on a monthly basis, one publication suggests the bid/spread bias issue has only a trivial impact on the small stock premium.

Some observers suggest that a delisting bias exists in the Morningstar decile size premium calculations due to its use of the CRSP database without adjustment.[28]

The reason for this possible bias is because the CRSP database information is allegedly missing prices for certain securities in the period immediately after these certain securities are delisted from a stock exchange.

According to the CRSP, as concluded in a CRSP white paper, the so-called delisting bias is greatly exaggerated.[29]

A few observers have suggested that the size effect is not relevant because various studies have ignored transaction costs in measuring rates of return.[30]

The primary observation is that, small capitalization stocks often have higher transaction costs than large stocks. Because of the higher transaction costs for small capitalization stocks, it is possible that the historical small-stock-related size premium would be reduced if transaction costs and holding periods were factored into the measurement of rates of return.

As published in the *Cost of Capital*, 5th edition, Ashok Abbott prepared a study of transaction costs by decile for securities listed on the NYSE, AMEX, and the Nasdaq from January 1993 to December 2008. The securities trading cost was estimated as the difference between the daily holding return (closing price to closing price) and the daily trading return (ask price from the previous day to the bid price of the current day).

As presented in Exhibit 2, as company size decreases, the average daily trading cost, as a percentage of the trade, increases. The study found that larger firms are traded at lower costs and are subject to less pricing pressure than smaller firms.

Abbott also prepared an analysis of trading costs as differentiated by liquidity. The results of the Abbott study suggest that as company liquidity decreases, trading costs increase. Another notable finding of the Abbott study indicates that the least liquid stocks comprise the smallest market capitalization size-related decile.

Exhibit 3 presents the Abbott study analysis of liquidity and trading costs.

Exhibit 2
Average Trading Costs by Market Value of Equity Decile
For the Period January 1993 to December 2008

Market Value of Equity Portfolio	Average Daily Trading Cost
1 - Largest Companies	0.75489%
2	1.07736%
3	1.33369%
4	1.67466%
5	2.05954%
6	2.50398%
7	3.16594%
8	4.13995%
9	5.57523%
10 - Smallest Companies	9.67356%

Source: *Cost of Capital* , 5th ed., 367.

A discussion of stock liquidity and the equity size premium is presented in more detail below.

Observations of the CRSP Size Premium 10th Decile Category

The companies that comprise the CRSP size premium 10th decile category have equity market capitalizations that range from $2.5 million to $262.9 million. As of December 31, 2016, the risk premium related to the companies comprising the 10th decile was 5.59 percent.[31]

Exhibit 3
Average Trading Costs Based on Equity Liquidity
For the Period January 1993 to December 2008

Decile by Liquidity	Average Daily Trading Cost
1 - Most Liquid Companies	1.48241%
2	1.82615%
3	2.02649%
4	2.15579%
5	2.28703%
6	2.47802%
7	2.73914%
8	3.03041%
9	3.73256%
10 - Least Liquid Companies	5.60277%

Source: *Cost of Capital* , 5th ed., 368.

The companies that comprise the CRSP size premium 10th decile are broken down into subcategories 10a and 10b, as presented in the Valuation Handbook. The companies that comprise the 10a subdecile include companies with market capitalizations between $127.3 million and $262.9 million, and the reported size premium is 4.09 percent.[32]

The companies that comprise the 10b subdecile include companies with market capitalizations between $2.5 million and $127.3 million, and the reported size premium is 8.64 percent.[33]

Within the 10a subdecile and 10b subdecile categories of the 10th decile, the Valuation Handbook presents more subcategories. The 10a subdecile is broken into 10w and 10x subdeciles, while the subdecile 10b is broken into 10y and 10z.

Exhibit 4 presents the Valuation Handbook, CRSP size premium 10th decile subdecile category market capitalizations and size premiums subcategory breakdown.

As provided in Exhibit 4, companies that are classified in the CRSP size premium 10th decile vary considerably in market capitalization and applicable size premium. The size premium ranges from 3.10 percent to 11.63 percent, a spread of 8.53 percent, or 853 basis points.

As seen in Exhibit 4, as the size of the company increases, its size premium risk decreases. That is why it is important to correctly interpret and apply the size premium component of the MCAPM—assuming an analyst applies an equity size premium adjustment.

According to the Valuation Handbook, "The CRSP Deciles Size Premia include all companies with no exclusion of speculative (e.g., start-up) or distressed companies whose market capitalization may be small because they are speculative or distressed."[34]

The distressed company issue can be seen through analysis of the 10th decile subcategories of 10y and 10z.

According to the *Valuation Handbook*, subdecile 10y includes companies in the 5th percentile with five-year average earnings before interest, taxes, depreciation, and amortization ("EBITDA") of negative $22.0 million. Companies classified in subdecile 10y at or below the 25th percentile (lower quartile) reported negative EBITDA.

Similarly, subdecile 10y companies have five-year net income ranging from negative $37.15 million to a positive $11.5 million. Not only are subdecile 10y companies significantly smaller, more than half are unprofitable.[35]

Exhibit 5 presents financial statistics related to the CRSP size premium 10th decile subcategories 10y and 10z as published in Valuation Handbooks for 2014 to 2017.

As presented in Exhibit 5, the companies that populate subcategory 10y and 10z are, on average, recording negative net income. In many cases, the companies that populate subcategory 10y and 10z are recording negative EBITDA.

Collectively, this information supports the theory that the CRSP size premium 10th decile is comprised of troubled and distressed companies.

According to James Hitchner in *Financial Valuation and Litigation Expert*, "It's important to note that 80 percent of the companies in decile category 10b are from 10z. As such, let's focus on 10z. At the 50th percentile of 10z the operating margin is –1.11 percent. Yes, on average, these companies are losing money. At the 25th percentile the operating margin is –21.27 percent. Furthermore, 62 percent of the companies in 10z are from only three industry sectors: financial services, technology, and healthcare."[36]

As indicated by Hitchner, based on dated information that is still relevant, not only does the CRSP size premium 10th decile include troubled companies, it is skewed by its industry concentration.

A few years back, Morningstar provided some additional detail related to the 10th decile regarding the probability of default of the companies in the 10th decile. Exhibit 6 provides statistics, as published in the Ibbotson *SBBI 2012 Valuation Yearbook* by Morningstar, of the probability of default of companies in the decile 10 subcategories.

Exhibit 4
10th Decile Subcategories
As of December 31, 2016

10th Decile Subcategory	Market Capitalization	Equity Size Premium
Decile 10w	$190.5 Million to $262.9 Million	3.10%
Decile 10x	$127.3 Million to $190.4 Million	5.33%
Decile 10y	$73.6 Million to $127.3 Million	7.21%
Decile 10z	$2.5 Million to $73.5 Million	11.63%

Source: *2017 Valuation Handbook: U.S. Guide to Cost of Capital* , Appendix 3.

Exhibit 5
10th Decile Subcategories 10y and 10z
Statistics as of September 30, 2013, 2014, 2015, and 2016

	Percent of Subcategory	Market Value of Equity (in $ Millions)	Market Value of Invested Capital (in $ Millions)	Sales (in $ Millions)	5-Year Average Net Income (in $ Millions)	5-Year Average EBITDA (in $ Millions)
As of September 30, 2013:	95th Percentile	181.19	566.53	734.63	12.99	80.76
10th Decile Subcategory 10y	75th Percentile	161.62	227.93	233.67	5.47	22.95
Market Value of Equity Range	**50th Percentile**	**138.58**	**175.02**	**74.86**	**(1.71)**	**7.74**
$100.9 Million and $184.9 Million	25th Percentile	116.69	139.05	29.38	(15.95)	(7.13)
	5th Percentile	103.44	110.39	1.42	(71.07)	(30.51)
As of September 30, 2013:	95th Percentile	94.04	210.99	318.61	7.56	27.73
10th Decile Subcategory 10z	75th Percentile	70.49	95.17	78.89	1.81	6.62
Market Value of Equity Range	**50th Percentile**	**44.97**	**64.98**	**31.77**	**(1.42)**	**1.18**
$2.4 Million and $100.8 Million	25th Percentile	25.12	34.97	15.29	(8.25)	(4.43)
	5th Percentile	7.89	11.23	1.03	(33.57)	(17.97)
As of September 30, 2014:	95th Percentile	184.30	916.88	848.90	12.99	104.79
10th Decile Subcategory 10y	75th Percentile	169.30	268.47	226.83	5.54	18.68
Market Value of Equity Range	**50th Percentile**	**153.44**	**182.31**	**72.73**	**(1.66)**	**5.47**
$116.3 Million and $190.5 Million	25th Percentile	133.02	158.54	32.72	(11.59)	(4.25)
	5th Percentile	118.02	127.99	2.15	(51.48)	(22.67)
As of September 30, 2014:	95th Percentile	108.54	216.73	254.92	7.31	22.16
10th Decile Subcategory 10z	75th Percentile	79.40	105.94	59.58	1.97	4.70
Market Value of Equity Range	**50th Percentile**	**53.00**	**72.10**	**26.04**	**(1.07)**	**0.04**
$3.0 Million and $115.9 Million	25th Percentile	30.78	39.48	10.00	(7.53)	(4.60)
	5th Percentile	11.52	15.44	0.21	(27.38)	(18.05)
As of September 30, 2015:	95th Percentile	106.48	574.94	638.20	15.03	90.47
10th Decile Subcategory 10y	75th Percentile	93.85	142.93	125.62	3.85	10.99
Market Value of Equity Range	**50th Percentile**	**81.81**	**99.61**	**41.82**	**(0.61)**	**2.22**
$64.8 Million and $108.6 Million	25th Percentile	73.90	82.50	22.04	(13.50)	(8.08)
	5th Percentile	65.82	68.54	8.36	(28.86)	(20.19)
As of September 30, 2015:	95th Percentile	61.95	231.27	321.69	5.63	29.72
10th Decile Subcategory 10z	75th Percentile	47.03	65.43	72.01	0.78	4.00
Market Value of Equity Range	**50th Percentile**	**32.09**	**43.61**	**27.47**	**(3.34)**	**(0.86)**
$2.0 Million and $64.7 Million	25th Percentile	17.65	24.99	9.81	(11.50)	(6.72)
	5th Percentile	6.13	9.35	1.79	(25.34)	(14.78)
As of September 30, 2016:	95th Percentile	123.59	694.33	516.09	11.54	69.39
10th Decile Subcategory 10y	75th Percentile	109.94	198.68	151.97	4.86	17.89
Market Value of Equity Range	**50th Percentile**	**96.02**	**121.77**	**51.50**	**(1.50)**	**3.99**
$73.6 Million and $127.3 Million	25th Percentile	82.85	99.80	29.23	(16.28)	(10.61)
	5th Percentile	74.68	77.79	8.28	(37.15)	(22.00)
As of September 30, 2016:	95th Percentile	70.11	176.78	248.60	4.60	22.77
10th Decile Subcategory 10z	75th Percentile	53.10	72.14	67.03	0.71	3.18
Market Value of Equity Range	**50th Percentile**	**34.34**	**46.75**	**25.30**	**(3.96)**	**(1.55)**
$2.5 Million and $73.5 Million	25th Percentile	18.85	25.49	8.09	(13.93)	(9.47)
	5th Percentile	6.66	9.76	1.03	(25.15)	(18.67)

Sources: *2017 Valuation Handbook: U.S. Guide to Cost of Capital* , Exhibit 4-10*; 2016 Valuation Handbook: Guide to Cost of Capital* , Exhibit 4-11*; 2015 Valuation Handbook: Guide to Cost of Capital* , Exhibit 4-11; and *2014 Valuation Handbook: Guide to Cost of*

Exhibit 6
Probability of Default of Decile 10 Subcategories
As of December 31, 2011

Probability of Default	10a Percent of Companies	10b Percent of Companies	10w Percent of Companies	10x Percent of Companies	10y Percent of Companies
75%	0	3	0	0	1
50%	2	7	1	3	3
25%	5	17	4	7	12
20%	6	21	4	7	14
15%	8	25	5	10	17
10%	10	31	8	13	22
5%	16	38	15	17	28

Source: *2012 Ibbotson SBBI Valuation Yearbook* , Table 7-15.

As of December 31, 2011, a little less than 20 percent of subcategory 10b had a 25 percent probability of default. As company size decreases, from subcategory 10w to subcategory 10z, the probability of default increases.

As presented in the Ibbotson *SBBI 2013 Valuation Yearbook* published by Morningstar, the 10th decile was comprised of significantly more companies in the 10b subcategory than the 10a subcategory.[37] As of December 31, 2002, there were 319 companies populating the 10a subcategory and 1,124 companies populating the 10b subcategory.

Furthermore, as of December 31, 2012, the significant majority of the 10b category was comprised of companies in the 10z subcategory—846 companies in 10z compared to 278 companies in 10y.[38]

Of these companies in the 10z subcategory, the majority were financial services businesses.[39]

Also, as presented in the *2013 Ibbotson SBBI Valuation Yearbook*, Morningstar changed its methodology for determining the likelihood of company default.

The results of the new methodology were similar to the results of the methodology used for *2012 Ibbotson SBBI Valuation Yearbook*. Morningstar concluded that financial distressed companies are more likely to be small equity capitalization stocks.[40]

Liquidity Issues That May Account for the Size Premium

According to Aswath Damodaran, "the notion that market for publicly traded stocks is wide and deep has led to the argument that the net effect of illiquidity on aggregate equity risk premiums should be small."[41]

It is generally accepted that less liquid securities are inherently of a greater risk profile than highly liquid securities and, therefore, investors require greater rates of return to invest in less liquid investments. In fact, a growing body of work investigating the impact of liquidity on returns has emerged.[42]

The cost of illiquidity on security pricing is influenced by macroeconomic direction. Stock illiquidity increases when economies slow down and during periods of crisis, thus exaggerating the effects of both phenomena on the equity risk premium.[43]

Security liquidity has value as discussed in the following example. Consider two assets with the same cash flows and average liquidity, but one asset has much more liquidity risk . . . if the assets had the same price, investors would avoid the one with the high liquidity risk, because they would fear bearing greater losses if they needed to sell it in a liquidity crisis.[44]

For many analysts, the calculation of the cost of equity includes a size premium alpha factor developed from the CRSP database. There are numerous theories addressing why small market capitalization stocks provide greater investment returns. However, there is an increasing amount of interest as to how the CRSP size premium decile conclusions may be skewed by an embedded liquidity discount.

Several studies have shown that an embedded stock liquidity discount helps to explain part of the reason that smaller capitalization companies generate higher returns—that is, the investor is compensated for investing in a low liquidity and therefore risker asset.

Exhibit 7 on the next page presents liquidity statistics and the impact of liquidity organized by equity market capitalization quartile classification. The analysis corresponds to publicly traded securities in the 1972 to 2016 time frame.

An interesting aspect of the embedded liquidity issue is that market capitalization and illiquidity are not always correlated since there are small, liquid companies and large, illiquid ones in the market.[45]

However, based on the data presented in Exhibit 7, it appears that the smallest capitalization securities are affected by liquidity concerns far more

than larger capitalization securities. It is also noteworthy that the subcategory of micro-cap stocks populated with the most companies, on average, was classified as low liquidity securities—a total of 342 companies.

In a research article published in the Journal of Business Valuation and Economic Loss, Frank Torchio and Sunita Surana studied the effect of liquidity on size premium calculations ("Torchio study").[46]

According to the Torchio study, a substantial portion of the size premium measurement reflects lack of liquidity. The Torchio study found that the lack of liquidity issue, an embedded liquidity issue, is problematic in certain fair value cases.

It is problematic because the application of the size premium—more specifically the application of the premium in small company valuations—may cause the fair value to be understated and may include an unintended valuation discount.

In order to study the effect of embedded liquidity related to the size premium, the Torchio study progressed through several procedures.[47] The three primary procedures are described as follows.

Exhibit 7
Liquidity Effect on Size Premium
Based on Quartile Portfolio Classifications
As Published in the 2017 *Valuation Handbook*

	Low Liquidity	Mid-Low Liquidity	Mid-High Liquidity	High Liquidity
Micro-Cap				
Geometric Mean (%)	16.03	15.66	9.65	-0.29
Arithmetic Mean (%)	18.41	19.28	14.97	4.91
Standard Deviation (%)	22.93	28.84	34.91	33.37
Average Number of Companies	342	182	125	98
Small-Cap				
Geometric Mean (%)	15.68	14.35	12.2	5.77
Arithmetic Mean (%)	17.35	16.93	15.52	9.96
Standard Deviation (%)	19.56	23.99	27.16	30.41
Average Number of Companies	198	201	174	175
Mid-Cap				
Geometric Mean (%)	14.19	13.9	12.67	8.15
Arithmetic Mean (%)	15.57	15.6	14.81	11.69
Standard Deviation (%)	17.75	19.68	21.75	27.63
Average Number of Companies	131	177	202	237
Large-Cap				
Geometric Mean (%)	11.2	12.05	11.75	8.89
Arithmetic Mean (%)	12.44	13.18	13.27	11.87
Standard Deviation (%)	16.33	15.5	17.76	24.79
Average Number of Companies	75	188	247	237

Source: *2017 Valuation Handbook: U.S. Guide to Cost of Capital* , Exhibit 4-13.

For the first procedure, the Torchio study replicated the Ibbotson SBBI 10 decile analysis using the CRSP database. The study applied the same or similar procedures used by Ibbotson, and now Duff & Phelps, to replicate the published SBBI 10 decile study results. It also replicated the 10th decile subcategories.

For the second procedure, the Torchio study subdivided the SBBI 10 deciles and 10th decile subcategories into high liquidity and low liquidity categories.

For the final procedure, the liquidity premium is calculated much the same way that the SBBI 10 decile size premiums are calculated. The liquidity premium is calculated as the excess return to the predicted CAPM return.

Exhibit 8 on the following page presents the Torchio study liquidity premium analysis results.[48]

The Torchio study provides empirical evidence of the impact that liquidity has on security rates of return. Based on Exhibit 8, the following conclusions appears to be true:

- The high liquidity level securities (stocks that exhibit trading liquidity above the decile group median) rates of return are significantly lower than the low liquidity level securities at each decile grouping.
- Compared to the size premium statistics presented in the *2011 Ibbotson SBBI Valuation Yearbook*, the high liquidity group for each decile and subdecile category had much lower rates of return.
- For SBBI deciles 1 through 9, the difference between the high liquidity equity premium estimate and the SBBI size premium is not as significant as it is for decile 10 and subcategories.
- The liquidity premium effect is most pronounced at the 10z subcategory decile.
- The size premium is clearly influenced by the low liquidity securities.

According to the Tochio study, the large-size premiums calculated by Ibbotson are the consequence of a disproportionately greater number of low liquidity stocks comprising the small-size portfolios.[49]

Exhibit 8
Liquidity Premium Analysis
Based on the Torchio Study
Using CRSP Data from 1926 to 2010

SBBI Decile Group	Liquidity Level	Liquidity Premium (return in excess of CAPM return) (%)	2011 Ibbotson SBBI Size Premium (%)	Difference between Liquidity and Size Premium (%)
1	High	-1.35	-0.38	**-0.97**
1	Low	0.13		0.51
2	High	-0.16	0.81	**-0.97**
2	Low	2.25		1.44
3	High	-0.05	1.01	**-1.06**
3	Low	2.88		1.87
4	High	0.07	1.20	**-1.13**
4	Low	3.25		2.05
5	High	0.57	1.81	**-1.24**
5	Low	4.01		2.20
6	High	-0.33	1.82	**-2.15**
6	Low	4.90		3.08
7	High	0.06	1.88	**-1.82**
7	Low	4.34		2.46
8	High	0.19	2.65	**-2.46**
8	Low	5.40		2.75
9	High	1.99	2.94	**-0.95**
9	Low	5.25		2.31
10	High	2.46	6.36	**-3.90**
10	Low	11.18		4.82
10w	High	-0.37	3.99	**-4.36**
10w	Low	8.08		4.09
10x	High	4.57	4.96	**-0.39**
10x	Low	10.40		5.44
10y	High	3.34	9.15	**-5.81**
10y	Low	12.85		3.70
10z	High	3.57	12.06	**-8.49**
10z	Low	17.55		5.49

Source: Frank Torchio and Sunita Surana, "Effect of Liquidity on Size Premium and its Implications for Financial Valuations," *Journal of Business Valuation and Economic Loss 9* , no. 1 (2014): Tables 10, 11, and 12.

For fair value in certain jurisdictions due to the presence of an embedded liquidity discount, the application of an equity size premium alpha factor based on the 10th decile or 10th decile subcategories may not be appropriate.

Certain Delaware Chancery Court Decisions Involving Size Premium Discussion

This section discusses certain appraisal rights actions filed in the Delaware Court of Chancery (the "Chancery Court"). The Delaware appraisal rights statute mandates fair value of a corporation as a going concern as the appropriate standard of value. The statute also allows the same fair value standard in shareholder oppression and shareholder appraisal rights actions to determine noncontrolling shareholder share value.

The Delaware Supreme Court clarified the meaning of fair value in 1950, defining it as the value that had been taken from the dissenting shareholder:

> The basic concept of value under the appraisal statute is that the stockholder is entitled to be paid for that which had been taken from him, viz., his proportionate interest in a going concern. By value of the stockholder's proportionate interest in the corporate enterprise is meant the true intrinsic value of his stock which has been taken by the merger.[50]

Recently, several Chancery Court decisions involved an equity size premium related discussion, including the following:

1. *Gearreald v. Just Care, Inc.* ("*Just Care*")
2. *Merlin Partners LP, and AAMAF, LP v. AutoInfo, Inc.* ("*Merlin Partners*")
3. *In re Appraisal of DFC Global Corp.* ("*DFC Global*")[51]
4. *Merion Capital L.P. and Merion Capital II L.P. v. Lender Processing Services, Inc.* ("*Merion Capital*")
5. *Dunmire v. Farmers & Merchants Bancorp of Western Pennsylvania, Inc.* ("*Dunmire*")

Just Care

In the *Just Care* decision, the treatment and application of the equity size premium was a significant point of contention. In *Just Care*, both experts agreed that, by size alone, the Just Care company should be classified in the Ibbotson decile category of 10b.

As of the valuation date, the Ibbotson 10b decile included companies with a market capitalization between $1.6 million and $136 million, and an indicated equity size premium of 9.53 percent. The specific point at issue is that the company expert applied the 9.53 premium while the petitioner's expert applied a smaller equity size premium of 4.11 percent.

The small equity size premium of 4.11 percent was based on the Ibbotson 10a decile. At trial it was argued that the 4.11 percent size premium was appropriate because of the "well-documented liquidity effect" contained within the size premium data.[52]

According to the petitioners' analyst, because "the illiquidity premium reflected in the size premium data for small cap stocks is akin to a liquidity discount" such a discount "must be eliminated in a fair value determination—much like a discount for lack of marketability or minority interest."[53]

In *Just Care*, the Chancery Court found that the petitioner's expert was correct in that a general liquidity discount cannot be applied in an appraisal rights proceeding. Such a discount generally relates to the marketability of the company's shares and is, therefore, prohibited.

In other words, on one hand, the Chancery Court found that entity or corporate level discounts were not appropriate and cited the *Borruso v. Communications Telesystems International* matter as support for its ruling.[54]

> To the extent Respondent is arguing for the application of a "corporate level" discount to reflect the fact that all shares of WXL shares were worth less because there was no public market in which to sell them, I read Cavalier Oil as prohibiting such a discount. This is simply a liquidity discount applied at the "corporate level." Even if taken "at the corporate level" (in circumstances in which the effect on the fair value of the shares is the same as a "shareholder level" discount) such a discount is, nevertheless, based on trading characteristics of the shares themselves, not any factor intrinsic to the corporation or its assets. It is therefore prohibited.

On the other hand, the Chancery Court found that although a liquidity discount related to the marketability of a company's shares is prohibited, that does not mean that the use of *any* input that is correlated with a company's illiquidity is, per se, invalid.[55]

A company's liquidity is highly correlated with its size, that is, smaller companies tend to be less liquid.[56] As a result, their equity is riskier and investors will demand higher returns from such investments, increasing the cost of capital. It is this kind of liquidity effect that is captured in the Ibbotson size premium.

In support of its decision, the Chancery Court cited the *JRC Acquisition Corp.* matter, as follows:[57]

> The Ibbotson size premium number reflects the empirical evidence that smaller firms have higher returns than larger firms. Petitioner's position that JR Cigar is a low-cap company (rather than a micro-cap company) decreases the expected rate of return on JR Cigar's stock by lowering the "size premium" applied. The problem with using liquidity as a basis for justifying a lower expected return, however, is that low liquidity is associated with higher expected returns. Investors seek compensation for the high transaction costs of illiquid securities, e.g., the bid/ask spread. In other words, even if JR Cigar had a higher market capitalization than the market price of its stock suggested because of its illiquidity, investors would still expect higher returns because of its illiquidity.

According to the Chancery Court, the liquidity effect, in this case, arises in relation to transactions between Just Care and its providers of capital.[58]

As such, the Chancery Court reasoned that, the liquidity effect is part of the company's value as a going concern. Where a company's illiquidity affects its ability to obtain financing for its operations, the company's overall risk and return profile will be affected, that is, the company will be worth less as a going concern because its financing costs are higher.[59]

In *Just Care*, the Chancery Court ruled against the petitioner theory that the embedded liquidity premium in the Ibbotson's size-related data should be adjusted in order to develop a cost of capital estimate.

The Chancery Court found that the liquidity effect at issue relates to the company's ability to obtain capital at a certain cost and not a shareholder level liquidity discount issue. This finding suggests

that the liquidity effect is related to a company's intrinsic value as a going concern, and it should be included when calculating its cost of capital.

The Chancery Court, in rejecting the petitioner argument, stated that the adjustment by the petitioners' analyst, as a matter of law, is unreliable. The Chancery Court concluded that small company-size premiums are regularly applied in appraisal proceedings in the Chancery Court without the type of adjustment performed by petitioners' analyst.

In other words, the Chancery Court found that the petitioner analyst's adjustment was unprecedented and, furthermore, had not been peer reviewed.

Although, the Chancery Court ruled against the petitioner argument in Just Care, it didn't completely dismiss the idea of a challenge. The Chancery Court concluded that it may adjust a company's size premium where sufficient evidence is presented to show that the company's individual characteristics make it less risky than would otherwise be implied under its corresponding decile classification based on size alone.

In the instant case, however, the petitioners' analyst did not argue that Just Care was less risky than other companies in decile 10b. The petitioners devoted only one sentence in the opening brief to attempt to justify the treatment of Just Care as a decile 10a company.[60]

Because petitioners did not provide compelling evidence for treating Just Care as a decile 10a company, the Chancery Court concluded that the decile 10b was appropriate based on the company size.

Merlin Partners

The *Merlin Partners* dispute is related to a cash-out merger of AutoInfo, Inc. ("AutoInfo"), shareholders. The petitioners in this matter demanded the appraisal of their shares in connection with the merger. Similar to the *Just Care* decision, the analyst for the petitioner and the analyst for the defendant did not agree on the appropriate equity size premium.

The petitioner analyst selected the size premium for Ibbotson's micro-cap category. The micro-cap category includes the 9th and 10th deciles. For the year of the instant case, companies in the 9th and 10th decile had market capitalizations ranging from $1.139 million to $514.209 million.[61]

The defendant analyst selected the 10z subdecile. The Ibbotson's 10z subdecile, at the time of the AutoInfo valuation analysis, consisted of companies with market capitalizations between $1.139 million and $96.164 million.[62]

At the time of the merger, AutoInfo had an approximate publicly traded market capitalization of $30 million.[63] Therefore, the AutoInfo market capitalization was fully within subdecile 10z. In *Merlin Partners*, because the AutoInfo market capitalization was within subdecile 10z, the Chancery Court concluded that AutoInfo should be classified in the 10z size premium and not the micro-cap size premium.

The petitioner valuation analyst testified that he "would have used [a size premium] close to the 10z category, if not 10z itself," had he not believed it necessary to strip out a marketability factor.[64]

The Chancery Court reasoned that, just like in *Just Care*, the petitioner's valuation adjustment ran counter to Delaware case law.

In the *Merlin Partners* decision, the Chancery Court rejected the adjustment of using a lower equity size-related premium (lower meaning smaller premium, due to the use of a higher capitalization decile classification) by reference to *Just Care* in the following passage: "because the liquidity effect at issue relate[d] to the Company's ability to obtain capital at a certain cost, . . . [and was therefore] related to the Company's intrinsic value as a going concern and should be included when calculating its cost of capital."[65]

The Chancery Court ruled that the publicly traded market capitalization of AutoInfo should be used to select the implied size-related equity risk premium.

DFC Global

The details related to the equity size premium issue in the *DFC Global* matter are different than the *Just Care* and *Merlin Partners* matters. However, the valuation analyst selection of the equity size premium was an issue in *DFC Global*.

Both analysts applied size premiums in calculating the DFC Global Corporation ("DGC") weighted average cost of capital.[66]

The analysts, however, disagreed on the magnitude of the equity size premium. The Chancery Court, in *DFC Global*, sided with the petitioner analyst's use of the publicly traded market capitalization of DGC in selecting an equity size premium.

Because DGC was publicly traded, the Court relied on the DGC equity market capitalization as of the date of the analysis. It then discounted the calculated equity market capitalization to account for the potential decrease in market capitalization due to discouraging financial results announced on the day of the DGC transaction.

In *DFC Global*, the Chancery Court considered that the defendant analyst arrived at a conclusion using a combination of the (1) the micro-cap premium and (2) the Duff & Phelps Risk Premium Report.

Because DGC was a financial-services-related business, the Chancery Court excluded the application of the Duff & Phelps Risk Premium Report for the subject business.[67]

The opposing analyst applied the 9th decile size premium. As of the valuation date, DGC had an approximate market capitalization of $346 million, which was in the 9th decile of $340 million to $633 million.[68]

Because the DGC market capitalization was near the lower end of the 9th decile and it had just announced poor financial performance that may not have been priced into the $346 million equity market capitalization, the Court selected the decile 10w size premium.[69]

One reason for selecting the 10w decile and not the micro-cap decile is that the decile 10w equity size premium is not as unduly influenced by very small companies and thinly traded stocks that are prevalent in the lower 10th decile equities.

Merion Capital

Merion Capital is a shareholder dispute related to a merger of Lender Processing Services, Inc. ("LPS"), and Fidelity National Financial, Inc. The petitioners demanded the appraisal of their shares in connection with the merger. Similar to the Just Care decision, the valuation analyst for the petitioner and valuation analyst defendant did not agree on the appropriate equity size premium. However in Merion Capital, one analyst applied a size premium and one did not.

In Merion Capital, the petitioner's analyst applied a 0.92 percent size premium.[70]

The respondent analyst did not add an equity size premium. The reason for not including an equity size premium was that there "is no consensus in the academic literature as to whether such a premium still exists."[71]

Because the respondent analyst did not add an equity size premium, and the exclusion of the size premium favored the petitioner, the Chancery Court accepted the respondent analyst decision not to add an equity size premium.

Dunmire

In *Dunmire*, the Chancery Court provided some commentary on the equity size premium issue in a footnote to its decision, as follows:[72]

> The use of a size premium is a subject of some controversy. *See, e.g., Guide to Cost of Capital* 4:8 ("In fact, some commentators contend that the historical data are so flawed that valuation analysts can dismiss all research results that support the size effect. For example, is the size effect merely the result of not measuring beta correctly? Are there market anomalies that simply cause the size effect to appear? Is size just a proxy for one or more factors correlated with size, suggesting that valuation analysts should use those factors directly rather than size to measure risk? Is the size effect hidden because of unexpected events?"); *see also* Hopkins Report Sections 138-45. I express no opinion on this debate. My use of a size premium simply follows from the fact that it is integral to the methodologies both experts utilized, from which my own determination of the discount rate is derived.

In *Dunmire*, both analysts used an equity size premium, so the Chancery Court did not take a formal position with respect to the equity size premium debate.

However, the Chancery Court's opinion suggested that it is open to considering arguments as to why the equity size premium may be excluded. It appears that the argument for and against the equity size premium is not likely to disappear anytime soon.

Summary and Conclusion

Analysts often use the income approach in valuation-related forensic analysis matters. The income approach may be used to estimate value in matters prepared according to the following standards of value:

- fair value
- fair market value
- intrinsic value
- investment value

There are at least two primary inputs to the income approach. The income stream or cash flow and the investment rate of return—present value discount rate—are primary components.

The focus of this discussion was to provide some background and information on the bits and pieces that form the foundation of the investment rate of return used to discount or capitalize the selected income stream.

Dating back to the Banz study, and more recently by way of the Duff & Phelps CRSP size premium analysis, empirical evidence has been gathered and analyzed in support of the size-related phenomena

theory. Small closely held company investment returns cannot be entirely explained by the standard application of the basic CAPM model for estimating the cost of equity capital.

Because the basic CAPM does not entirely explain small closely held company investment returns, analysts typically apply the MCAPM to estimate the cost of equity capital in such instances.

There are many observations regarding the size-related phenomena theory and the CRSP size premium data used by a majority of analysts. These observations include the following:

1. The small capitalization premium has disappeared in recent years. The empirical evidence supports varying size-related premium at different points in time. Therefore, in certain time periods, it would not be surprising for small capitalization stocks to provide lower investment returns than larger capitalization stocks.
2. Premium, at the smallest level, is unduly influenced by stocks of less than $5 million in market capitalization and stocks that trade at prices less than $2 per share. The most statistical noise in the CRSP size premium data is in the 10th decile classification and its smaller subcategory classifications. This factor may not be as relevant if the subject matter company is a very small business that is similar to the companies that populate the 10th subcategories of 10y and 10z.
3. The idea that other factors, specifically liquidity or lack thereof, provide important detail that analysts should consider in the decision to use, or not to use, the CRSP size premium data.

If the valuation assignment is a fair value matter, the analyst should consider research that is intended to illustrate the explanatory factors behind the size premium phenomena. Based on the Torchio study results and liquidity analysis presented in the *Valuation Handbook*, the CRSP size premium data may incorporate an embedded liquidity discount factor.

By using the CRSP size premium data—specifically the for the 10th decile category—an analyst may be incorporating an unintended discount into the valuation assignment. If the embedded liquidity theory holds, the incorporation of an embedded liquidity discount may, at some point, run counter to Delaware Court of Chancery case law regarding fair value.

But for now, the application of the implied CRSP size premium to develop a cost of equity is a generally accepted business and security valuation practice.

Notes:

1. There are many other cost of equity capital estimation models including (a) the Duff & Phelps, LLC, Risk Premium Report Model; (b) arbitrage pricing theory model; and (c) Fama-French three factor model.
2. CRSP is an acronym for Center for Research in Security Prices. The *Valuation Handbook* is a continuation of the previously produced *SBBI Valuation Yearbook* by Morningstar. The *Valuation Handbook* is produced by Duff & Phelps.
3. Unsystematic risk is defined as the portion of total risk that is specific to an individual security and can be avoided through diversification. Shannon P. Pratt, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 5th ed. (New York: McGraw-Hill, 2008), 1075, Appendix A.
4. CAPM is defined as a model in which the cost of capital for any stock or portfolio of stocks equals a risk-free rate plus a risk premium that is proportionate to the systematic risk of the stock or portfolio. Pratt, *Valuing a Business*, 1070, Appendix A.
5. Beta is defined as a measure of the systematic risk of a stock; the tendency of a stock's price to be correlated with changes in a specific index. Pratt, *Valuing a Business*, 1070, Appendix A.
6. Roger J. Grabowski, "The Size Effect—It Is Still Relevant," *Business Valuation Review* 35, no. 2 (Summer 2016): 63.
7. Rolf W. Banz, "The Relationship between Return and Market Value of Common Stocks," *Journal of Financial Economics* 9 (March 1981): 3–18.
8. Duff & Phelps, *2017 Valuation Handbook: U.S. Guide to Cost of Capital* (New York: John Wiley & Sons, 2017), 2–14.
9. The *Valuation Handbook* presents an alternative size premium analysis, the Risk Premium Report. The Risk Premium Report is not discussed herein.
10. Annual stock market returns represent the combined annual stock returns of stocks listed on the New York Stock Exchange ("NYSE"), NYSE Euronext, and Nasdaq.
11. The standard deviation is a measure that is used to quantify the amount of variation or dispersion of a set of data values. A low standard deviation indicates that the data points tend to be close to the mean of the set, while a high standard deviation indicates that the data points are spread out over a wider range of values. J.M. Bland and D.G. Altman, "Statistics Notes: Measurement Error," *The BMJ* 312 (7047) (September 1996): 1654.
12. Aswath Damodaran, "Equity Risk Premiums (ERP): Determinants, Estimation and

Implications—The 2015 Edition," Stern School of Business whitepaper (March 2015): 37.

13. Ibid.
14. Aswath Damodaran, "The Small Cap Premium: Where Is the Beef?" *Business Valuation Review* 34, no. 4 (Winter 2015): 153.
15. Joel L. Horowitz, Tim Loughran, and N.E. Savin, "The Disappearing Size Effect," *Research in Economics* 54, no. 1 (2000): 87.
16. Ibid.
17. Ibid., 96.
18. Duff & Phelps, *2017 Valuation Handbook: U.S. Guide to Cost of Capital*, 4–6.
19. Grabowski, "The Size Effect—It Is Still Relevant": 65.
20. Ibid.
21. Damodaran, "The Small Cap Premium: Where Is the Beef?": 154.
22. Horowitz, Loughran, and Savin, "The Disappearing Size Effect."
23. Damodaran, "The Small Cap Premium: Where Is the Beef?": 154.
24. Ibid.
25. Roger Grabowski and Shannon Pratt, *Cost of Capital*, 5th ed. (New York: John Wiley & Sons, 2014), 363.
26. Horowitz, Loughran, and Savin, "The Disappearing Size Effect": 87.
27. Grabowski and Pratt, *Cost of Capital*, 364.
28. Ibid., 365.
29. Ibid.
30. Ibid., 366.
31. Duff & Phelps, *2017 Valuation Handbook: U.S. Guide to Cost of Capital*, Appendix 3.
32. Ibid.
33. Ibid.
34. Ibid., 4–12.
35. Ibid., 4–13.
36. Jim Hitchner, "How to 'Rig' a Valuation: The Discount Rate," *Financial Valuation and Litigation Expert* (February/March 2013).
37. *2013 Ibbotson SBBI Valuation Yearbook* (Chicago: Morningstar, 2013), 90.
38. Ibid.
39. Ibid., 91.
40. Ibid., 100.
41. Damodaran, "Equity Risk Premiums (ERP): Determinants, Estimation and Implications—The 2015 Edition": 12.
42. Duff & Phelps, *2017 Valuation Handbook: U.S. Guide to Cost of Capital*, 4–21.
43. Damodaran, "Equity Risk Premiums (ERP): Determinants, Estimation and Implications—The 2015 Edition": 12.
44. Yakov Amihud, Haim Mendelson, and Lasse Heje Pedersen, *Market Liquidity, Asset Pricing, Risk, and Crises* (Cambridge: Cambridge University Press, 2013), 103.
45. Damodaran, "The Small Cap Premium: Where Is the Beef?"
46. Frank Torchio and Sunita Surana, "Effect of Liquidity on Size Premium and Its Implications for Financial Valuations," *Journal of Business Valuation and Economic Loss* 9, no. 1 (2014): 55–85.
47. The Tochio study was based on monthly stock data provided by the CRSP database for the period of 1926 to 2010.
48. Torchio and Surana, "Effect of Liquidity on Size Premium and its Implications for Financial Valuations": 77–79.
49. Ibid.: 77.
50. Tri-Continental v. Battye, 74 A.2d 71, 72 (Del. 1950).
51. In re Appraisal of DFC Global Corp., C.A. No. 10107-CB, 2016 WL 3753123 (Del. Ch. July 8, 2016), *rev'd and rem'd sub nom.* DFC Global Corporation v. Muirfield Value Partners, L.P., No. 518, 2016, 2017 WL 3261190 (Del. Aug. 1, 2017).
52. Gearreald v. Just Care, Inc., C.A. No. 5233-VCP, 2012 WL 1569818 at *10 (Del. Ch. Apr. 30, 2012).
53. Id.
54. Id. at *11, citing Borruso v. Communications Telesystems International, 753 A.2d 451, 460 (Del. Ch. 1999).
55. Id. at *10
56. Id. at *11
57. Id., citing Cede & Co. v. JRC Acquisition Corp., No. Civ.A. 18648-NC, 2004 WL 286963 at *9 (Del Ch. Feb. 10, 2004).
58. Id.
59. Id.
60. Id. at *12.
61. Merlin Partners LP and AAMAF, LP, v. AutoInfo, Inc., C.A. No. 8509-VCN, 2015 WL 2069417 at *15 (Del. Ch. Apr. 30, 2015).
62. Id.
63. Id.
64. Id.
65. Id.
66. In re Appraisal of DFC Global Corp., 2016 WL 3753123 at *12.
67. The Duff & Phelps Risk Premium Report size-related statistics exclude financial services businesses.
68. In re Appraisal of DFC Global Corp., 2016 WL 3753123 at *12.
69. Id. at *14.
70. Merion Capital L.P. and Merion Capital II L.P., v. Lender Processing Services, Inc., C.A. No. 9320-VCL, 2016 WL 7324170 at *29 (Del. Ch. Dec. 16, 2016).
71. Id.
72. Dunmire v. Farmers & Merchants Bancorp of Western Pennsylvania, Inc., C.A. No. 10589-CB, 2016 WL 6651411, n.139 (Del. Ch. Nov. 10, 2016).

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