The Discount for Lack of Marketability: Update on Current Studies and Analysis of Current Controversies

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I. Introduction

The difference in price an investor will pay for a liquid asset compared to a comparable illiquid asset is often substantial. This difference in price is commonly referred to as the "discount for lack of marketability" (DLOM). That is, the DLOM measures the difference in the expected price between (1) a liquid asset (that is, the benchmark price measure) and (2) an otherwise comparable illiquid asset (that is, the valuation subject). The measurement of the appropriate DLOM continues to be a controversial topic, particularly with regard to valuations performed for gift and estate tax, shareholder litigation, buy-sell agreement, and family law purposes.

This Article first summarizes the concepts of investment liquidity and illiquidity (that is, the conceptual basis for the DLOM) (Parts I and II), the empirical studies and the theoretical models that are commonly used to estimate the DLOM (Parts III, IV, and V) and the application of the DLOM to a closely held business valuation (Part VI). Then the Article analyzes the factors that influence the magnitude of the DLOM (Part VII) and the current controversies regarding DLOM analyses (Part VIII).

A. *The Concept of Investment Liquidity*

The liquidity of a security ownership interest relates to how quickly and certainly the security can be converted into cash at the owner's discretion. Investors value liquidity. Rational investors will pay a price premium for liquidity. Rational investors will also demand a price discount for the lack of liquidity. Fundamentally, the security of a closely held company is not as liquid as an

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otherwise comparable security of a publicly traded company. That is, a closely held company security does not have the same degree of marketability as an otherwise comparable publicly traded security.

The terms *marketability* and *liquidity* are sometimes used interchangeably. However, for purposes of this Article, references to marketability (or lack of marketability) will relate to the owner's ability (or inability) to sell an individual security or a block of securities. Also, for purposes of this Article, references to liquidity (or illiquidity) will relate to the owner's ability (or inability) to sell a business enterprise or a controlling ownership interest in a business enterprise.

There are varying degrees of investment marketability. In fact, there is a spectrum of investment marketability, ranging from (1) fully marketable to (2) fully nonmarketable. An ownership interest of an actively traded security can typically be converted into cash within three business days of the sell decision. This is the typical investment benchmark for a fully marketable security. At the other end of the investment marketability spectrum is an ownership interest in a privately owned business entity that pays no dividends and makes no other distributions, requires capital contributions, and limits ownership of the company to certain individuals. Of course, there exists a range of marketability positions in between these two extremes.

B. *Reasons to Apply a Valuation Adjustment*

In the United States public capital markets, a security holder can sell an actively traded security over the telephone in seconds, usually at or within a small fraction of a percent of the last price at which a security traded. These public market transactions occur at a very small commission cost. And, the investor will typically receive the transaction proceeds following a Federally mandated settlement period of three business days. By contrast, the population of potential buyers for most closely held investment securities is a small fraction of the population of potential buyers for publicly traded investment securities. In fact, it is illegal for a person or a company to sell closely held securities to the general public without first registering the security offering with either the Securities and Exchange Commission (SEC) or the state corporation commission. Such a security offering registration is an expensive and time-consuming process. Furthermore, a noncontrolling stockholder cannot register closely held corporation shares for public trading; only the closely held corporation itself can register its shares for public trading.

Besides the problems associated with selling closely held stock, it is also difficult for investors to hypothecate closely held stock. That is, the value of closely held shares is further impaired by the unwillingness of banks and other lending institutions to accept such securities (as they would accept public stock) as loan collateral.

Because of these contrasts between the ability to sell or hypothecate closely held stock as compared to publicly traded shares, empirical evidence suggests that the DLOM for closely held corporation securities tends to cluster in the

range of 25 to 50% (compared to comparable publicly traded securities). Of course, each nonmarketable security should be analyzed on the basis of its individual facts and circumstances, which may justify a DLOM above or below this typical range of price adjustments.

C. *Baseline from Which to Apply the DLOM*

In the valuation of privately owned businesses and security interests, analysts typically apply some combination of the three generally accepted business valuation approaches: (1) the market approach, (2) the income approach, and (3) the asset-based approach. Depending on the individual valuation variables used, the market approach and income approach conclude value indications on either a controlling ownership interest basis or a noncontrolling ownership interest basis. As it is commonly applied, the asset-based approach concludes a value indication on a controlling ownership interest basis. The three generally accepted business valuation approaches typically conclude value indications on a marketable ownership interests basis. In this closely held security context, the degree of marketability in the value indication is influenced by the individual variables used in each valuation approach or method. Therefore, value indications are as marketable (or as nonmarketable) as the market-derived empirical evidence on which the valuation analyses are based. The magnitude of the subject DLOM will depend on the facts and circumstances related to (1) the subject closely held business and (2) the subject nonmarketable security.

Security-specific and company-specific factors are not the only reason to apply a DLOM. Certain engagement-specific factors may also affect the magnitude of the DLOM. One engagement-specific factor is the appropriate level of value for the valuation. The next two sections discuss the DLOM in the context of two common levels of value: (1) the noncontrolling ownership interest level of value and (2) the controlling ownership interest level of value.

D. *DLOM for a Noncontrolling Ownership Interest*

There is a continuous spectrum of value influences with regard to investment marketability. This continuous spectrum of value influences ranges from (1) absolute control and perfect liquidity (for example, equal to the liquidity of actively traded stock listed on a public stock exchange) to (2) absolute lack of control and completely illiquid (for example, imposed by contractual restrictions that limit the potential buyers and dictate the sale price). It is impossible to describe all of the discrete steps along the ownership control-marketability continuum. However, these two investment attributes represent a continuous spectrum of combined valuation adjustments.

The generally accepted business valuation approaches can result in a value indication that is stated on either (1) a controlling ownership interest basis or (2) a noncontrolling ownership interest basis. For example, the market approach-guideline merged and acquired company method generally con-

cludes a value indication on a controlling ownership interest basis. Alternatively, the market approach-guideline publicly traded company method generally concludes a value indication on a noncontrolling ownership interest basis. Therefore, when the valuation objective is to estimate value on a nonmarketable, noncontrolling ownership interest basis, a DLOM may be appropriate. In addition, the application of a discount for lack of control (DLOC) may also be appropriate, depending on which valuation methods (and valuation variables) the analysts used.

With the exception of the "cost to obtain liquidity" studies, the empirical studies summarized in this Article are based on applying a DLOM to a noncontrolling ownership interest. In contrast, the cost to obtain liquidity studies apply when the subject security interest is a controlling ownership interest. It is noteworthy that the DLOM empirical studies discussed here are based on securities trading at different levels on the ownership control-marketability spectrum. Therefore, the analyst should consider and compare the marketability of the subject security to the marketability of the securities analyzed in the various DLOM empirical studies. This concept of comparing relative marketability (and other investment attributes) is repeated throughout this Article.

E. *Illiquidity for a Controlling Ownership Interest*

Closely held controlling ownership interests suffer from illiquidity in somewhat the same way as closely held noncontrolling ownership interests. Marketability is an investment attribute the analyst should consider when valuing one percent of the shares of a closely held company. Likewise, liquidity is an investment attribute that an analyst should consider when valuing 100% of the shares of a closely held company. The investment attribute is based on the principle that the marketability of an ownership interest—whether controlling or noncontrolling—is determined by the ability of the owner to quickly, and with some degree of certainty, convert the ownership interest to cash.

There is consensus among analysts that consideration of a DLOM is appropriate when valuing both noncontrolling ownership interests and controlling ownership interests in nonpublic companies. The Tax Court validated this consensus in *Estate of Andrews*: "[E]ven controlling shares in a nonpublic corporation suffer from lack of marketability because of the absence of a ready private placement market and the fact that flotation costs would have to be incurred if the corporation were to publicly offer its stock."[1]

Subsequent judicial decisions affirmed the appropriateness of applying a DLOM to the valuation of a controlling ownership interest.[2] Of course, this

[1]Estate of Andrews v. Commissioner, 79 T.C. 938, 953 (1982).

[2]*See, e.g.*, Estate of Dunn v. Commissioner, 79 T.C.M. (CCH) 1337, 2000 T.C.M. (RIA) ¶ 2000-012; Estate of Maggos v. Commissioner, 79 T.C.M. (CCH) 1861, 2000 T.C.M. (RIA) ¶ 2000-129; Estate of Jameson v. Commissioner, 77 T.C.M. (CCH) 1383, 1999 T.C.M. (RIA) ¶ 99,043; Estate of Dougherty v. Commissioner, 59 T.C.M. (CCH) 772, 1990 T.C.M. (P-H) ¶ 90,274.

application is a function of both (1) the valuation methods and variables used by the analyst and (2) the subject valuation level of value. This control interest DLOM is due to the following two factors: (1) the absence of a ready private placement market and (2) flotation costs, which would be incurred in achieving liquidity through a public offering. The owner faces the following transaction risk factors when attempting to liquidate a controlling equity interest:

1. An uncertain time horizon to complete the offering or sale;
2. "Make ready" accounting, legal, and other costs to prepare for and execute the offering or sale;
3. Risk as to the eventual sale price;
4. Uncertainty as to the form (for example, stock or cash) of transaction sale proceeds;
5. Inability to hypothecate the subject equity interest; and
6. Investment banker or other brokerage fees.

Risk factors one through five are summarized next. Risk factor six (investment banker or brokerage fees), is summarized in Part IV.C.

F. *Uncertain Time Horizon Risk*

In some instances, it takes months (or even years) to complete the offering or sale of the private corporation controlling block of stock. Clearly, this uncertain (but considerable) time horizon is in direct conflict with the principle of marketability. The principle of marketability implies a very short equity-interest-for-cash conversion period.

G. *Make Ready Cost Risk*

As discussed in Part IV.C., there can be substantial costs to (1) prepare the company for sale and (2) execute the offering or sale. A study in 2005 concluded that underwriter costs alone typically represent seven percent of the deal size in an initial public offering (IPO).[3] This figure does not include other transaction costs, which include (1) auditing and accounting fees to provide potential buyers the financial information and assurances they demand; (2) legal costs to draft the necessary documents, to clear potential contingent liabilities, and to negotiate warranties; and (3) administrative costs on the part of the business owners to deal with the accountants, lawyers, or potential buyers, or all of their representatives. In *The Cost of Going Public,* Jay Ritter estimated these "other" transaction make-ready costs to be between 2.1 percent and 9.6 percent of the IPO amount.[4]

[3]Hsuan-Chi Chen & Jay R. Ritter, *The Seven Percent Solution,* 55 J. FIN. 1105, 1129 (2000).

[4]Jay Ritter, *The Costs of Going Public,* 19 J. FIN. ECON. 269-81 (1987).

H. *Sale Price Risk*

There is a risk that the selling security holder will not achieve his or her expected sale price. This failure to realize the expected sale price can result from many factors, including:

1. Overstatement of the business or security valuation on which the expected price is based;
2. Occurrence of company-specific events during the market exposure period that caused the sale price to decrease;
3. Occurrence of market-specific events during the market exposure period that caused the sale price to decrease;
4. Lack of receptivity by capital markets to stocks of companies in the subject industry; or
5. Lack of receptivity by capital markets to the subject company.

I. *Sale Proceeds Risk*

The definitions of marketability and liquidity assume an asset is sold for cash. If some or all of the sale proceeds is in a form other than cash, then the cash-equivalency transaction price may be less than the reported transaction consideration. Examples of sale proceeds components that may have a cash equivalency value less than face value include (1) restricted public stock, (2) seller-provided below-market financing, (3) consideration that is based on future contingency payments, and (4) consideration that is based on future earnout payments.

J. *Inability to Hypothecate Risk*

Banks are often reluctant to lend money collateralized by an equity interest in a privately owned company. While awaiting a sale, if the owner of a controlling ownership interest needs cash, it may be impossible to borrow against the value of the business interest. This inability to hypothecate occurs even if the ownership interest represents 100% (or absolute control) of the closely held company equity.

II. Standard of Value

The standard of value, which is determined by the valuation objective, is an important consideration in the DLOM. For example, fair market value analyses may have different DLOM considerations than fair value analyses. Some of the different standards of value include (1) fair market value, (2) fair value, (3) investment value, (4) intrinsic value, (5) owner value, (6) use or user value, (7) acquisition value, and (8) other. The appropriate standard of value is typically determined by the purpose and objective of the valuation engagement.

A. *Fair Market Value Standard of Value*

One common standard of value is *fair market value.* Valuation analysts sometimes rely on the definition included in Revenue Ruling 1959-60[5] (and also in regulation section 20.2031-1(b)).[6] Revenue Ruling 1959-60 defines fair market value as "the price at which the property would change hands between a willing buyer and a willing seller when the former is not under any compulsion to buy and the latter is not under any compulsion to sell, both parties having reasonable knowledge of relevant facts."

The willing buyer and the willing seller in this standard of value are hypothetical persons, rather than specific individuals or entities. In the fair market value context, the DLOM may be an important consideration in the valuation of privately owned company securities. An informed hypothetical buyer of a privately owned company will consider all investment characteristics of the subject company, including lack of marketability.

B. *Fair Value Standard of Value*

Business and security valuations related to dissenting stockholder appraisal rights statutes or shareholder oppression statutes often apply the fair value standard of value. There is no generally accepted definition of "fair value" for purposes of these statutory appraisal matters because the definition often relates to the statutory authority, judicial precedent, and administrative rulings in the state in which the action is litigated. The statutory and judicial authorities in many states imply (or specify) that fair value is estimated before consideration of any level of value discounts. In these jurisdictions, a DLOC and a DLOM may not apply. In other words, fair value is estimated on a marketable, controlling ownership interest level of value. Analysts estimating the fair value of closely held stock for purposes of such statutes should understand the legal definition of fair value appropriate to the particular engagement.

III. Analytical Models to Quantify the DLOM

When a DLOM is applicable, analysts often rely on two types of models to quantify the appropriate adjustment: (1) empirical models and (2) theoretical models. Generally, empirical models use analyses based on empirical capital market transaction observations, rather than on theoretical economic principles. In contrast, theoretical models do not rely on actual capital market pricing evidence, but are based on fundamental microeconomic relationships.

A. *Empirical Models*

Empirical models rely on actual capital market transactions to provide evidence for estimating a DLOM. There are two common categories of empirical capital market studies:

[5]Rev. Rul. 1959-60, 1959-1 C.B. 237.
[6]Reg. § 20.2031-1(b).

1. Studies of price discounts on sales of restricted shares of publicly traded companies (the restricted stock studies); and
2. Studies of price discounts on private stock sale transactions prior to an initial public offering (the pre-IPO studies)

B. *Theoretical Models*

Theoretical models, unlike empirical models, do not derive their DLOM conclusions from actual capital market transaction data. There are two categories of theoretical models: (1) option pricing models and (2) discounted cash flow models.

IV. DLOM Empirical Models

A. *The Restricted Stock Studies*

Privately owned companies may raise capital by completing a private placement of debt or equity securities. In an equity private placement, a company can issue either registered stock or unregistered (that is, restricted) stock to an accredited investor. Registered stock includes the shares of publicly traded companies that can be freely traded in the open market. In contrast, unregistered shares of stock are not registered for trading on a stock exchange. Therefore, unregistered shares cannot be freely traded in the open market. When publicly traded companies issue restricted (unregistered) stock, the restricted stock is typically sold at a price discount compared to the price of the (registered) publicly traded stock. Companies are willing to accept a price discount on the sale of their restricted stock because the time and cost of registering the new stock with the SEC would make the stock issuance or capital formation impractical. These observed price discounts (that is, public stock price compared to same company private stock price) indicate a DLOM. And, these stock price discount data are the basis for the restricted stock studies discussed below.

Securities Exchange Commission (SEC) Rule 144[7] governs the purchase and sale of stock issued in unregistered private placements. According to the SEC, "[w]hen you acquire restricted securities or hold control securities, you must find an exemption from the SEC's registration requirements to sell them in the marketplace. Rule 144 allows public resale of restricted and control securities if a number of conditions are met."[8]

These conditions mentioned in SEC Rule 144 relate to: (1) investment holding period, (2) adequate current information, (3) trading volume formula, (4) ordinary brokerage transactions, and (5) filing of a notice with the SEC The investment holding period restrictions on the transfer of restricted

[7]17 C.F.R. § 230.144 (revised April 1, 1990).

[8]U.S. Securities & Exchange Commission, Rule 144: Selling Restricted and Control Securities, *available at* http://www.sec.gov/investor/pubs/rule144.htm (last visited Nov. 15, 2007).

stock eventually lapse, usually after 12 months.[9] At that point, the trading volume formula is typically the most restrictive sale condition of SEC Rule 144. The trading volume formula allows the subject securities to be "dribbled out" in the marketplace. Depending on the size of the block of the subject securities, the dribble out formula may require the investor to sell small portions of the subject securities over a multi-year period. Rather than dribble out the sale of the restricted securities, the owner of restricted stock can sell his or her securities in a privately negotiated transaction, subject to sections 4(1) and 4(2) of the Securities Act of 1933.[10]

Until 1995, restricted stock sale transactions had to be reported to the SEC and thus were a matter of public record. Since 1995, analysts have collected restricted stock sale transaction data from private sources. Therefore, there is available a body of data on the prices of private transactions in restricted securities. These transaction price data can be used for comparison with prices of the same company unrestricted securities eligible for trading on the open market.

SEC Institutional Investor Study

In an SEC study of institutional investor actions, one of the topics was the amount of the price discount at which transactions in restricted stock took place.[11] This price discount was measured by comparing private transaction prices to the prices of identical but unrestricted stock on the open market.[12] The SEC study concluded an average price discount of 24% for all transactions of restricted stock—that is, a price discount to their unrestricted counterpart stock. In addition, the study concluded generally higher price discounts for restricted stocks that would trade on the over-the-counter market (OTC) once the restrictions expired. These higher price discounts were compared to stocks that would trade on the New York Stock Exchange (NYSE) or the American Stock Exchange (ASE). The average price discount for stocks that would trade on the OTC when the restrictions expired was approximately 35%.

One of the outcomes of the SEC Institutional Investor Study was SEC Accounting Series Release (ASR) No. 113, dated October 21, 1969,[13] and No. 118, dated December 23, 1970.[14] ASR No. 113 and No. 118 require

[9]On February 18, 1997, the SEC adopted amendments to reduce the holding period requirements under Rule 144 of the Securities Act from two years to one year for the resale of limited amounts of restricted securities. The amendment became effective April 29, 1997.

[10]Securities Act of 1933 § 4(1)-(2), 15 U.S.C. § 77d(1)-(2) (2000).

[11]INSTITUTIONAL INVESTOR STUDY REPORT OF THE SECURITIES AND EXCHANGE COMMISSION, H.R. Doc. No. 92–64, at 2444–56 (1971).

[12]*Id.*

[13]Accounting Series Release No. 113, Investment Company Act Release No. 5847, 35 Fed. Reg. 19,989 (Dec. 31, 1970).

[14]Accounting Series Release No. 118, Investment Company Act Release No. 6295, 35 Fed. Reg. 19,986 (Dec. 31, 1970).

investment companies registered under the Investment Company Act of 1940 to disclose their policies for the cost and valuation of their restricted securities. The result of this disclosure requirement was that an ongoing body of empirical pricing data became available. These data included the relationship between restricted stock prices and same company freely traded stock prices. Several analysts have analyzed these data to provide empirical benchmarks for estimating the DLOM.

Gelman Study

In 1972, Milton Gelman published the results of his study of the prices paid for restricted securities by four closed-end investment companies specializing in restricted securities investments.[15] Based on an analysis of 89 restricted stock sale transactions between 1968 and 1970, Gelman concluded that both the mean and median price discounts were 33%. Gelman also concluded that almost 60% of the restricted stock sales indicated discounts of 30% or higher.

Trout Study

In a study of restricted stock purchases by mutual funds from 1968 to 1972, Robert Trout constructed a financial model to provide an estimate of the price discount appropriate for a private company's stock.[16] Trout's multiple regression model was based on pricing data from 60 restricted stock transactions. The Trout multiple regression analysis concluded an average price discount of approximately 33.5% for restricted stock (compared to the same company freely traded counterpart stock). Like the SEC study, Trout concluded that companies with stocks listed on national exchanges experienced lower price discounts on their restricted stock transactions than did companies with stocks traded over-the-counter.

Moroney Study

In 1973, Robert E. Moroney published the results of his study of the prices paid for restricted securities by ten registered investment companies.[17] The study analyzed 146 restricted stock purchases. The average price discount was approximately 35.6% and the median price discount was approximately 33.0%. Moroney contrasted the empirical evidence of the actual sale transactions with the lower average DLOM conclusions allowed in most prior court decisions on gift and estate tax matters. He pointed out, however, that the empirical evidence on the prices of restricted stocks was not available as

[15]Milton Gelman, *An Economist-Financial Analyst's Approach to Valuing Stock of a Closely-Held Company*, 36 J. TAX'N. 353 (1972).

[16]Robert R. Trout, *Estimation of the Discount Associated with the Transfer of Restricted Securities*, 55 TAXES 381 (1977).

[17]Robert E. Moroney, *Most Courts Overvalue Closely Held Stocks*, 51 TAXES 144 (1973).

a benchmark for quantifying the DLOM at the time of these prior court cases. And, Moroney suggested that higher DLOM adjustments be allowed in future judicial decisions now that the relevant empirical pricing data are available.

Maher Study

J. Michael Maher published the results of another restricted stock study in 1976.[18] The Maher study was similar to the Moroney study in that it compared prices paid for restricted stocks with the market prices of the same company unrestricted stock. He concluded that mutual funds were not purchasing restricted securities during 1974 and 1975, which were recessionary years in general for the stock market. Therefore, the empirical data that Maher actually used covered the five-year period from 1969 through 1973. He concluded that the mean DLOM for the years 1969 through 1973 was 35.43%.[19] He further eliminated the top and bottom ten percent of the restricted stock purchases in an effort to remove especially high- and low-risk situations. When the "outlier" data were removed, the result of this study was almost identical with a mean price discount of 34.73%.

Standard Research Consultants Study

In 1983, Standard Research Consultants (SRC) analyzed private placements of restricted common stock to test the then-current applicability of the above-described SEC study.[20] SRC studied 28 private placements of restricted common stock from October 1978 through June 1982. The price discounts concluded in the SRC study ranged from seven percent to 91%, with a median price discount of 45%.

Willamette Management Associates Study

Willamette Management Associates (WMA) analyzed private placements of restricted stock for the period of January 1, 1981, through May 31, 1984. The early part of this unpublished study overlapped the last part of the SRC study. However, relatively few restricted stock sale transactions took place during the overlap period. Most of the restricted stock sale transactions in the WMA study occurred in 1983.

WMA identified 33 transactions during that period that could reasonably be classified as arm's-length and for which the price of the restricted shares could be compared with the price identical but unrestricted shares of the same company. The median discount for the 33 restricted stock transactions

[18]J. Michael Maher, *Discounts for Lack of Marketability for Closely Held Business Interests*, 54 TAXES 562 (1976).

[19]*Id.* at 571.

[20]William F. Pittock & Charles H. Stryker, *Revenue Ruling 77–287 Revisited*, 10 STANDARD RES. CONSULTANTS 1 (1983).

compared to the prices of their freely tradable counterparts was 31.2%. The slightly lower average percentage discounts for private placements during this time may be attributable to the somewhat depressed prices in the public stock market, which in turn reflected the recessionary economic conditions prevalent during most of the period of the study.

Hertzel & Smith Study

Michael Hertzel and Richard L. Smith (Hertzel & Smith) conducted a restricted stock study covering 106 private placements occurring between January 1, 1980, and May 31, 1987. The average sale proceeds in the private placement transaction sample equaled $11.4 million, and the average equity market value of the private placement transaction sample equaled $94.7 million.[21]

Hertzel & Smith conducted their study primarily to test whether private placement price discounts reflect a resolution of asymmetric information about the subject company value. With regard to information effects, Hertzel & Smith found evidence that price discounts reflected the costs incurred by private investors to assess the firm value. In other words, companies that are difficult to analyze (for example, privately owned companies and companies with significant intangible assets), received a larger DLOM compared to DLOMs received by companies that are more easily analyzed (for example, publicly traded companies that are followed by equity analysts).

The Hertzel & Smith study showed an average price discount for the private placements of 20.14%. They noted:

> Private placement discounts in our sample vary widely. 39 of the placements in our sample were made at discounts of more than 25% and eight were at premiums of more than ten percent. Premiums appear to reflect the value of control, cash infusions by investors who already own much of the outstanding stock, and market price declines between the time the placement price is negotiated and when it is announced to the market. Discounts appear to be related to such factors as resale restrictions, placement size, and type of investor.[22]

They analyzed the difference in the price discount between (1) the private placements of restricted shares and (2) the private placement of unrestricted shares, concluding: "We find an additional discount of 13.5% for placements of *Restricted shares.*"[23] Additional discussion of this study appears in Part VIII.

Silber Study

In 1991, William L. Silber presented the results of his analysis of 69 private

[21]Michael Hertzel & Richard L. Smith, *Market Discounts and Shareholder Gains for Placing Equity Privately*, 48 J. FIN. 459, 470 (1993).
[22]*Id.* at 470-71.
[23]*Id.* at 470.

placements of public corporation common stock between 1981 and 1988.[24] Silber concluded that the average price discount was 33.75%, a conclusion that is consistent with earlier restricted stock studies. Silber also concluded that the size of the price discount tended to be higher for private placements that were larger, as measured as a percentage of the shares outstanding. Silber found a small effect on the price discount on the basis of the size of the company, as measured by revenue. "Tripling the revenues from the sample mean of $40 million to $120 million increases the relative price of the restricted shares by only 2.9 points (from 71.7 to 74.6)."[25]

Bajaj, Denis, Ferris, and Sarin Study

This study (the Bajaj study) analyzed 88 private placements that occurred between January 1, 1990, and December 31, 1995.[26] The Bajaj study separately analyzed (1) registered private placement issues and (2) unregistered private placement issues. The authors concluded, based on their study, that (1) the average price discount for unregistered issues was 28.1%, (2) the average price discount for registered issues was 14.0%, and (3) the overall average price discount was 22.2%.

The authors of the Bajaj study hypothesized that the observed private placement price discounts were due to factors other than illiquidity. Specifically, they attributed the observed price discounts to the following four factors: (1) the fraction of total shares offered in the private placement; (2) business risk, as measured by price volatility in the issuer's publicly traded shares; (3) financial distress, as measured by Altman's Z-score; and (4) total transaction proceeds from the private placement. "[C]ontrolling for all other factors influencing private placement discounts, an issuer would have to concede an additional discount of 7.23% simply to compensate the buyer for lack of marketability."[27] An additional analysis of this study is presented in Part VIII.

Johnson Study

Bruce Johnson conducted a restricted stock study that attempted to provide a link between (1) the wide range of price discounts observed in previous restricted stock studies and (2) subject company characteristics.[28] Johnson analyzed 72 restricted stock transactions that occurred between 1991 and 1995. The average price discount reported in the Johnson study was 20%,

[24]William L. Silber, *Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices*, 47 FIN. ANALYSTS J. 60 (1991).

[25]*Id.* at 64.

[26]Mukesh Bajaj, David J. Denis, Stephen P. Ferris, & Atuyla Sarin, *Firm Value and Marketability Discounts*, 27 J. CORP. L. 89 (2001).

[27]*Id.* at 114.

[28]Bruce Johnson, *Restricted Stock Discounts, 1991–95*, 5 SHANNON PRATT'S BUS. VALUATION UPDATE 1, 2 (Mar. 1999).

or 22% after excluding bank and real estate investment trust companies. The subject company factors considered in the Johnson study were (1) current year net income, (2) previous year net income, (3) current year revenue, and (4) transaction size. According to Johnson, "an appraiser should consider some of the factors that influence the magnitude of the discount which include profitability, size, holding period and transaction amount."[29]

Management Planning Study

Management Planning, Inc. performed a study titled "Analysis of Restricted Stocks of Public Companies: 1980–1996."[30] This study analyzed 53 restricted stock sale transactions. In selecting the 53 transaction samples, Management Planning eliminated potential transactions based on the following factors: (1) companies that suffered a loss in the fiscal year preceding the private transaction; (2) companies defined as "start-up" companies (that is, companies having revenue of less than $3 million); and (3) stock transactions that were known to have registration rights. Their study found an average price discount of 27%.

Daniel McConaughy, David Cary, and Chao Chen expanded the Management Planning study by analyzing factors that contributed to the reported price discounts.[31] The underlying data analyzed in this study were the same transactions analyzed in the Management Planning study (that is, the 53 private placements of restricted stock that occurred during 1980 to 1996). This study shows that the discounts required by investors are positively related to business risk, financial risk, market risk, and degree of illiquidity. The two more important factors are business risk, in the form of size in sales and consistent historical sales growth, which are associated with lower discounts; and market risk, in the form of stock price volatility, which is associated with higher discounts.[32]

FMV Study

FMV Opinions, Inc. examined restricted stock transactions from 1980 through 1997.[33] This analysis of 243 transactions resulted in a mean price discount of approximately 22.1% and a median price discount of 20.1%. The FMV study also examined the relationship between (1) the observed price

[29]*Id.* at 3.

[30]Robert P. Oliver & Roy H. Meyers, *Discounts Seen in Private Placements of Restricted Stock: The Management Planning, Inc., Long-Term Study (1980-1996), in* HANDBOOK OF ADVANCED BUSINESS VALUATION 97 (Robert F. Reilly & Robert P. Schweihs eds., 2000).

[31]Daniel L. McConaughy, David Cary, & Chao Chen, *Factors Affecting Discounts on Restricted Stock*, 4 VALUATION STRATEGIES 14 (2000).

[32]*Id.* at 16.

[33]Espen Robak, *FMV Introduces Detailed Restricted Stock Study*, 7 SHANNON PRATT'S BUS. VALUATION UPDATE 1, 3 (Nov. 2001).

discount and (2) several descriptive factors. These descriptive transactions included: the industry, trading market (exchange traded or over-the-counter), the percentage size of the block, and the total market value of the subject corporation. According to the FMV study, the DLOM was higher for over-the-counter traded securities than securities traded on a large national exchange. This relationship is particularly relevant for the shares of closely held companies, which typically are more comparable to shares traded on over-the-counter exchanges.

FMV has provided periodic updates to its initial study. Currently, the FMV study includes 205 transactions that occurred up to March 2005.[34] The FMV study concludes that certain company-specific factors affect the magnitude of the DLOM. Analysis of the companies in the FMV DLOM database reveals certain relationships. Table 1 illustrates these relationships by separating the FMV study companies into quintiles based on the observed price discount.

Table 1
FMV Restricted Stock Study Results: Characteristics of High Price Discount vs. Low Price Discount Transactions

Quintile:	1	2	3	4	5
Percentage Price Discount	1.0%	11.7%	20.8%	31.5%	47.4%
Market Value ($0)	133,470	89,689	66,172	57,286	31,175
Total Assets ($0)	46,200	27,874	16,758	10,725	6,878
Share Price Volatility	73%	74%	73%	85%	110%
Price Per Share ($)	11.20	9.59	7.06	6.48	4.56

The most notable trend that can be observed from the above table is that larger price discounts are present in smaller, more highly volatile companies. This trend is corroborated by the various other restricted stock studies summarized in this discussion. Another important finding of the FMV study is that block size is directly correlated with price discount.

Columbia Financial Advisors, Inc. Studies

Columbia Financial Advisors, Inc. (CFAI) conducted a study of the sale of restricted securities over the period January 1, 1996, through April 30, 1997.[35] The restricted stock sale transactions were identified from the Securities Data Corporation U.S. New Issues private placement database. A total of 123 private placements were included in this database for the selected time period. A total of 100 transactions were eliminated for various reasons, leaving 23 transactions included in the CFAI study. The observed price discounts ranged from 0.8 to 67.5% for all 23 transactions, with an average DLOM of

[34]The FMV study is available as a searchable database at www.bvmarketdata.com (last visited Jan. 14, 2008).

[35]Kathryn F. Aschwald, *Restricted Stock Discounts Decline as Result of 1-Year Holding Period,* 6 SHANNON PRATT'S BUS. VALUATION UPDATE 1, 3 (May 2000).

approximately 21% and a median DLOM of 14%.

These observed price discounts are generally lower than the price discounts recorded in the earlier studies discussed above. One explanation for this decrease is the increase in volume of privately placed stock (SEC Rule 144(a)) in the period studied. During the CFAI study observation period, it was generally known that the SEC-required holding period was scheduled to change from two years to one year in 1997.

Using the same methodology and sources, CFAI conducted a second study.[36] The second study attempted to identify the impact of the increased liquidity resulting from the change in the SEC-required holding period. CFAI examined common equity private placements during the period January 1, 1997, through December 31, 1998. There were a total of 270 common stock private placements during that time. A total of 255 transactions were eliminated for various reasons, leaving 15 transactions for the study. The price discounts ranged from zero to 30%, with an average DLOM of approximately 13% and a median DLOM of nine percent. These price discounts are generally lower than the price discounts observed in the earlier CFAI study. This result is most likely due to the change in the SEC-required holding period from two years to one year.

LiquiStat Study

The LiquiStat study, conducted by Pluris Valuation Advisors (Pluris), was published in the January/February 2007 edition of *Valuation Strategies*.[37] Pluris identified two weaknesses with prior restricted stock studies: (1) the lack of measurable parameters with regard to the price discount (for example, was the observed price discount the result of company size or information asymmetry between the buyer and the seller?) and (2) the impossibility of establishing two distinct data sets, one completely liquid and one completely illiquid. Pluris reasoned that the observed price discounts from previous restricted stock studies were likely affected by factors unrelated to illiquidity, such as: (1) compensation for control and monitoring, (2) capital scarcity effects, and (3) information asymmetry effects.

To overcome these perceived weaknesses, Pluris analyzed the pricing of restricted stock in investor-to-investor trades—that is, transactions (1) not involving the issuer or an affiliate of the issuer and (2) not raising new capital for the issuer. According to Espen Robak of Pluris,

> Clearly, the private placement process has facets, beyond just illiquidity, that affect discounts. The solution, or at least part of the solution, might be to take a look at the pricing of restricted stock in investor-to-investor trades, not involving the issuer or an affiliate of the issuer and not raising capital for the issuer.[38]

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[36] *Id.* at 4.

[37] Espen Robak, *Lemons or Lemonade? A Fresh Look at Restricted Stock Discounts*, 10 VALUATION STRATEGIES 4 (2007).

The data analyzed was from the LiquiStat database of private sales transactions created by Pluris. As of the date of the Pluris study, the database contained transactions facilitated by Restricted Stock Partners through its Restricted Stock Trading Network. There were 61 transactions analyzed in the LiquiStat study. The 61 transactions analyzed in the LiquiStat study were completed at an average price discount of 32.8%, and a median price discount of 34.6%. The average number of days remaining before the shares sold became available to trade in the public markets was 144 days.

Restricted Stock Study Conclusions

These 17 restricted stock transaction studies cover several hundred transactions spanning the late 1960s through 2006. The results of these various restricted stock studies are summarized in Table 2 below:

Table 2
Restricted Stock Studies
Summary of Observed Price Discounts

Restricted Stock Study	Observation Period of Study	Observed Average Price Discount
SEC Overall Average	1966–69	25.8%
SEC Nonreporting OTC Companies	1966–69	32.6%
Milton Gelman	1968–70	33.0%
Robert R. Trout	1968–72	33.5%
Robert E. Moroney	1969–72	35.6%
J. Michael Maher	1968–73	35.4%
Standard Research Consultants	1978–82	45.0%
Willamette Management Associates	1981–84	31.2%
Hertzel & Smith	1980–87	13.5%
William L. Silber	1981–88	33.8%
Baja, Denis, Ferris, and Sarin [a]	1990–95	22.2%
Johnson Study	1991–95	20.0%
Management Planning, Inc.	1980–96	27.0%
FMV Opinions, Inc.[b]	1980–97	22.1%
Columbia Financial Advisors, Inc.	1996–97	21.0%
Columbia Financial Advisors, Inc.	1997–98	13.0%
LiquiStat	2005–06	32.8%

[a] This study attributes price discount to factors other than marketability (for example, compensation for the cost of assessing the quality of the firm and for the anticipated costs of monitoring the future decisions of its managers.

[b] Represents results of latest published study. The database is routinely updated and available for purchase at www.bvmarketdata.com.

First, these restricted stock studies generally conclude a decline in the average DLOM after 1990. The restricted stock transactions analyzed in the studies covering the period from 1968 through 1988 (where the average DLOM was approximately 35%) were generally less marketable than the restricted stocks analyzed after 1990 (where the average DLOM ranged between 20% and 25%). Valuation analysts typically attribute the decline in observed price

[38]Espen Robak, *Discounts for Illiquid Shares and Warrants: The LiquiStat Database of Transactions on the Restricted Securities Trading Network* (Pluris Valuation Advisors White Paper), Jan. 22, 2007, at 13-14, *available at* http://www.plurisvaluation.com/pressroom/liquistat-wp.pdf.

discounts to (1) the increase in volume of privately placed stock under SEC Rule 144(a) and (2) the change in the minimum SEC-required holding period under Rule 144—from two years to one year—that took place as of April 29, 1997. Increased volume was the result of a Rule 144 amendment in 1990 that allowed qualified institutional investors to trade unregistered securities amongst themselves. By increasing the potential buyers of restricted securities, the marketability of these securities generally increased.

Both of these explanations suggest that investors have not changed the way they value liquidity. Rather, the liquidity of restricted securities has increased. As it became easier to find a buyer for restricted securities after 1990, the average restricted stock price discount decreased. The same trend occurred after the SEC holding period decreased from two years to one year in 1997. As the market for restricted stocks has become more liquid, it is increasingly important for analysts to consider the facts and circumstances of each restricted stock study. Specifically, it is important to compare the market for the subject closely held company with the market for restricted securities. If the expected holding period for the stock in a closely held companies is two years or greater, it may be more meaningful to select a DLOM based on the restricted stock studies conducted prior to 1990 (in addition to considering other theoretical and empirical research). Alternatively, if the subject closely held stock is likely to be liquidated within one year, the post-1990 studies may be more meaningful.

Second, the restricted stock studies indicate the wide range in observed price discounts within each study. Although the average price discounts calculated in the restricted stock studies is similar, the range of price discounts observed in each of the studies was quite large, ranging from a price premium to price discounts approaching 90%.

The wide range of price discounts has two implications. First, a large range could lower the reliability of the data. Second, and more importantly, it suggests that valuation analysts should use professional judgment to select a price discount for a given subject company based on these data. The most likely explanation for the wide range in observed price discounts is the myriad of company-specific and security-specific factors that affect the DLOM. Therefore, while a DLOM is clearly indicated from the restricted stock studies, it is up to the individual analyst to consider how the subject interest relates to the restricted stock average price discount observed in the restricted stock studies.

Restricted shares of public corporation stock may not (temporarily) be traded directly on a stock exchange. However, the investor has certainty that, in a relatively short time period, the trading restrictions will lapse. The shares of stock of a closely held corporation, on the other hand, may never be traded directly on a stock exchange. The prospect of any level of efficient marketability is much lower for closely held companies shares compared to restricted public company shares. Therefore, the appropriate DLOM related to closely

held corporation shares (or to similar closely held investment securities) is generally considered to be greater than the DLOM indicated by restricted stock studies.

B. *Pre-IPO Studies*

The second type of empirical analysis that quantifies the appropriate DLOM for closely held stock is the pre-IPO study. A pre-IPO study examines arm's-length sale transactions in the stock of a closely held company that has subsequently achieved a successful initial public offering of its stock. In a pre-IPO study, the DLOM is quantified by analyzing (with various adjustments) the difference between (1) the public market price at which a stock was issued at the time of the IPO and (2) the private market price at which a stock was sold (in an arm's-length transaction) prior to the IPO. There are three published pre-IPO studies, summarized below.

Emory Studies

A number of studies were conducted under the direction of John D. Emory, currently president of Emory Business Valuation, LLC, in Milwaukee, Wisconsin.[39] The pre-IPO studies covered various time periods from 1980 through 2000.[40] The basic methodology for the various pre-IPO studies was identical. The population of companies in each study consisted entirely of initial public offerings during the respective period in which Baird & Company either participated in or received prospectuses. The prospectuses of these 4,088 offerings were analyzed to determine the relationship between (1) the price at which the stock was initially offered to the public and (2) the price at which the latest private transaction took place up to five months prior to the initial public offering.

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[39]Mr. Emory was formerly with Robert W. Baird & Co., where the studies prior to April 1997 were conducted.

[40]John D. Emory Sr., F.R. Dengel III, & John D. Emory Jr., *The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock May 1997 through December 2000*, 20 BUS. VALUATION REV. 15-19 (2001); John D. Emory, *The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock (Eighth in a Series) November 1995 through April 1997*, 16 BUS. VALUATION REV. 123-31 (1997); John D. Emory, *The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock (Seventh in a Series) January 1994 through June 1995*, 14 BUS. VALUATION REV. 155-60 (1995); John D. Emory, *The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock February 1992 through July 1993*, 13 BUS. VALUATION REV. 3-5 (1994); John D. Emory, *The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock August 1990 through January 1992*, 11 BUS. VALUATION REV. 208-12 (1992); John D. Emory, *The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock February 1989 through July 1990*, 9 BUS. VALUATION REV. 114-16 (1990); John D. Emory, *The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock (Aug. 1987–Jan. 1989)*, 8 BUS. VALUATION REV. 55-57 (1989); John D. Emory, *The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock, January 1985 through June 1986*, 5 BUS. VALUATION REV. 12-15 (1986); John D. Emory, *The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock—January 1980 through June 1981*, 4 BUS. VALUATION REV. 21-24 (1985).

Excluding the results of the 1980–81 study, which were uncharacteristically high, the mean price discount was in the range of 42 to 48% and the median price discount was in the range of 40 to 45%. The fact that these average price discounts are a little more than ten percentage points greater than the average price discounts indicated by the above-mentioned restricted stock studies is predictable. The pre-IPO stock sale transactions occurred when there was not yet any established secondary market for the subject stock. Table 3 summarizes the results of each of the Emory pre-IPO studies.

Table 3
Emory Pre-IPO Studies
Indicated DLOM Results

Pre-IPO Study	Number of Prospectuses Reviewed	Number of Qualifying Transactions	Indicated Price Discount Mean	Indicated Price Discount Median
1980–1981	97	13	60%	66%
1985–1986	130	21	43%	43%
1987–1989	98	27	45%	45%
1989–1990	157	23	45%	40%
1990–1991	266	35	42%	40%
1992–1993	443	54	45%	44%
1994–1995	318	46	45%	45%
1995–1997	732	91	43%	42%
1997–2000	1,847	36	48%	44%

In September 2000, Emory and his associates published a similar pre-IPO study that dealt with dot-com companies only.[41] This pre-IPO study covered the period from May 1997 through March 2000. This study analyzed a total of 53 sales transactions, 42 of which were convertible preferred stock transactions and 11 of which were common stock transactions. For the 53 transactions, the study calculated an average price discount of 54% and a median price discount of 54%. For the 42 convertible preferred stock transactions, the study calculated a 54% average price discount and a 53% median price discount. For the 11 common stock transactions, the study concluded an average price discount of 54% and a median price discount of 59%.

Valuation Advisors Studies

Valuation Advisors, LLC (VA), maintains a database that includes over 2,400 pre-IPO transactions that occurred within two years of an IPO. The pre-IPO transactions are arranged into five time periods: four three-month intervals for the 12 months immediately before an IPO, and a single period for the timeframe from one to two years before the IPO. The pre-IPO transactions are also arranged by type of security (that is, stock, convertible preferred stock, or option).

[41]John D. Emory Sr., F.R. Dengel III, & John D. Emory Jr., *The Value of Marketability as Illustrated in Initial Public Offerings of Dot-Com Companies May 1997 through March 2000*, 19 BUS. VALUATION REV. 111-21 (2000).

VA performed a pre-IPO DLOM study for each of the years 1999, 2000, 2001, and 2002. The conclusion of the VA 1999 pre-IPO study indicated an average one-year DLOM of 51.91%.[42] In the 2000 pre-IPO study, the average one-year DLOM was concluded to equal 47.07%.[43] The average one-year DLOM concluded in the VA 2001 pre-IPO study equaled 22.41%.[44] However, the average DLOM equaled 40.84% when a narrowed DLOM range of ten to 90% was analyzed. This narrowed DLOM range was considered to reduce the influence of (1) cheap stock or stock options and (2) price premiums due to changing stock market conditions. Table 4 below summarizes the results of the VA pre-IPO studies.

Table 4
Valuation Advisors Pre-IPO Study
Indicated DLOM Results

	Period Before IPO in Which Transaction Occurred					
IPO Year	0-3 Months	4-6 Months	7-9 Months	10-12 Months	1-2 years	Number of Transactions
1999	30.8%	54.2%	75.0%	76.9%	82.2%	695
2000	28.7%	45.1%	61.5%	68.9%	76.6%	653
2001	14.7%	33.2%	33.4%	52.1%	51.6%	115
2002	6.2%	17.3%	21.9%	39.5%	55.0%	61

Source: Shannon P. Pratt. "Using Pre-IPO and Restricted Stock Data to Estimate Discounts for Lack of Marketability." *Business Valuation Review*, March 2004.

As indicated in the above table, the DLOM observed in 2001 and 2002 was lower than the DLOM observed in the prior years studied.

Willamette Management Associates Studies

Willamette Management Associates (WMA) completed 18 pre-IPO DLOM studies covering the period of 1975 through 1997.[45] WMA also completed an additional pre-IPO DLOM study encompassing the five years 1998 through 2002.[46] As in the previous pre-IPO studies, the 1998–2002 pre-IPO study included only private market stock sale transactions that were conducted on an arm's-length basis. The transactional data analyzed in the 1998–2002 pre-IPO study included (1) sales of closely held corpora-

[42]Brian K. Pearson, 1999 Discount for Lack of Marketability Study, *available at* http://valuationpros.com/ipo_1999.html (last visited Jan. 14, 2008).

[43]Brian K. Pearson, 2000 Marketability Discounts as Reflected in Initial Public Offerings (2001), *available at* http://valuationpros.com/ipo_2000.html (last visited Jan. 14, 2008).

[44]Brian K. Pearson, The 2001 Marketability Discount Study (2002), *available at* http://www.valuationpros.com/ipo.html (last visited Jan. 14, 2008).

[45]Shannon P. Pratt, Business Valuation Discounts and Premiums 84-85 (2001).

[46]Pamela Garland and Ashley Reilly, *Update on the Willamette Management Associates Pre-IPO Discount for Lack of Marketability Study for the Period 1998 Through 2002*, Insights (Willamette Management Associates) (Spring 2004).

tion stock in private placements and (2) repurchases of treasury stock by the closely held corporation. All transactions involving the granting of employee, executive, or other compensation-related stock options were eliminated from consideration in the 1998–2002 study. All transactions involving stock sales to corporate insiders or other related parties were eliminated from consideration in the 1998–2002 study. These transactions were eliminated from considerations unless WMA could verify (by a telephone interview with at least one principal party) that the stock sale transaction was, in fact, a bona fide, arm's-length transaction.[47]

The results of the WMA studies are presented in Table 5. The average price discounts varied from period to period. However, in most cases, the average price discounts concluded in the WMA pre-IPO study were greater than the average price discounts concluded in the restricted stock DLOM studies. The difference is most likely due to the fact that—unlike pre-IPO transactions—restricted stock transactions involve companies that already have an established public trading market.

Table 5
WMA Pre-IPO Studies
Summary of Price Discounts for Private Transaction P/E Multiples
Compared to Initial Public Offering P/E Multiples
Adjusted for Changes in Industry P/E Multiples

Time Period Analyzed	Number of Companies Analyzed	Number of Transactions Analyzed	Standard Mean Price Discount	Trimmed Mean Price Discount*	Median Price Discount	Price Discount Standard Deviation
1975–78	17	31	34.0%	43.4%	52.5%	58.6%
1979	9	17	55.6%	56.8%	62.7%	30.2%
1980–82	58	113	48.0%	51.9%	56.5%	29.8%
1983	85	214	50.1%	55.2%	60.7%	34.7%
1984	20	33	43.2%	52.9%	73.1%	63.9%
1985	18	25	41.3%	47.3%	42.6%	43.5%
1986	47	74	38.5%	44.7%	47.4%	44.2%
1987	25	40	36.9%	44.9%	43.8%	49.9%
1988	13	19	41.5%	42.5%	51.8%	29.5%
1989	9	19	47.3%	46.9%	50.3%	18.6%
1990	17	23	30.5%	33.0%	48.5%	42.7%
1991	27	34	24.2%	28.9%	31.8%	37.7%
1992	36	75	41.9%	47.0%	51.7%	42.6%
1993	51	110	46.9%	49.9%	53.3%	33.9%
1994	31	48	31.9%	38.4%	42.0%	49.6%
1995	42	66	32.2%	47.4%	58.7%	76.4%
1996	17	22	31.5%	34.5%	44.3%	45.4%
1997	34	44	28.4%	30.5%	35.2%	46.7%
1998	14	21	35.0%	39.8%	49.4%	43.3%
1999	22	28	26.4%	27.1%	27.7%	45.2%
2000	13	15	18.0%	22.9%	31.9%	58.5%
2001	2	2	-195.8%	NA	-195.8%	NA
2002	5	7	55.8%	NA	76.2%	42.8%

*Excludes the highest and lowest deciles of indicated discounts.

NA = Not Applicable

Source: Pamela Garland & Ashley Reilly, *Update on the Willamette Management Associates Pre-IPO Discount for Lack of Marketability Study for the Period 1998 Through 2002*, INSIGHTS (Willamette Management Associates) (Spring 2004).

[47]SHANNON P. PRATT, ROBERT F. REILLY, & ROBERT P. SCHWEIHS, VALUING A BUSINESS: THE ANALYSIS AND APPRAISAL OF CLOSELY HELD COMPANIES 408-11 (4th ed. 2000) (detailing the specific analytical procedures performed in the various WMA pre-IPO DLOM studies) (hereinafter PRATT ET AL).

The DLOM conclusions for 1999 and 2001 are significantly lower than the DLOM conclusions of the previous WMA pre-IPO studies. In addition, the DLOM conclusions for 1999 and 2001 are significantly lower than (1) the DLOM conclusions reported in the VA study for the same years, and (2) the DLOM conclusions reported in the Emory study. There are several capital market reasons why the 1999 and 2000 results of the WMA DLOM study are outside the range of results observed in the 1975 through 1998 period: (1) there were relatively few IPO companies and relatively few private sale transactions that qualified for inclusion in the WMA pre-IPO study in 1999 and 2000, (2) the height of the dot-com "bubble" occurred during this time frame, and (3) the average first-day returns for IPO stocks were extraordinarily high in 1999 and 2000.[48]

In addition, the WMA DLOM conclusions for 2001 appear to be unreasonable. Certain capital market factors affected the observed price discounts. More importantly, however, is the fact that the DLOM conclusions for these years are based on an extremely small number of transactions. Based on these factors, analysts should either (1) rely on the collective results of the 5-year period between 1998–2002 as a reasonable indication of the DLOM, or (2) ignore the results from the 2001 and 2002 period.

Pre-IPO Study Conclusions

The evidence from the pre-IPO DLOM studies is compelling. The pre-IPO studies cover hundreds of transactions during a span of over 20 years. Median price differences between private transaction prices and public market prices varied under different market conditions, ranging from about 40 to 60%, after eliminating the "outliers." Analysts agree that pre-IPO DLOM studies provide a relevant empirical data with regard to the DLOM for a privately owned company. The reason is that companies in the pre-IPO DLOM studies more closely resemble privately held companies to which the DLOM is being applied. The pre-IPO DLOM studies are the only DLOM studies that involve transactions in shares of privately owned companies.

Whether a valuation analyst is examining a company, ownership interest, or transaction, it is important that the subject interest be as similar as possible to the data used in the analysis. In this regard, if the subject interest is a privately owned security with an expected holding period exceeding two years, then the pre-IPO DLOM studies may provide a more relevant comparison than the restricted stock DLOM studies. The unique factors of each company and each engagement will affect which specific DLOM studies should be analyzed. Two major criticisms of the pre-IPO DLOM studies are: (1) selection bias and (2) the fact that IPO prices are inflated due to "hype."

[48]Robert F. Reilly, *Willamette Management Associates' Discount for Lack of Marketability Study for Marital Dissolution Valuations*, 19 AM. J. FAM. L. 44, 48-49 (2005) (explaining these uncharacteristic results).

First, the selection bias argument is based on the fact that only successful companies complete an IPO. Pre-IPO DLOM studies eliminate from consideration, by definition, companies that filed for an IPO but were unsuccessful. If there was a bias based on the fact that the pre-IPO DLOM studies include only "successful" companies, it would understate the size of the DLOM. One would expect a "troubled" company to be less liquid than a "successful" company, with fewer options for liquidity resulting in a greater DLOM. In addition, the impact of selection bias on the indicated average DLOM may be minimal. This is because only about one-in-five companies that file for an IPO actually fail to complete the IPO when scheduled. WMA conducted a failed-IPO study that compared (1) the number of companies that filed an IPO registration with the SEC on Form S-1 to (2) the number of companies that successfully completed their IPO.[49] The WMA failed-IPO study found that, among other things, (1) from 1990 through 2002, approximately 8,100 companies filed IPO registration statements with the SEC, and (2) approximately 1,800, or 23.3%, of those companies did not complete the IPO. The WMA failed-IPO study considered a registration to be "failed" if an IPO was not completed within 18 months of the IPO registration. Some companies may have completed an IPO after this 18-month period, thereby lowing the percentage of failed IPOs. Based on the relatively low percentage of failed IPOs indicated by the WMA failed-IPO study, this factor may have a minimal effect on the reported average DLOM.

Second, the price inflation hype argument is based on the theory that underwriters over-hype new issues. Therefore, underwriters drive IPO stock prices up and increase the indicated DLOM. However, empirical studies generally find first-day stock price appreciation on IPOs are often substantial, indicating that IPOs are systematically underpriced. According to Tim Loughran and Jay Ritter, "In the 1980s, the average first-day return on initial public offerings (IPOs) was seven percent. The average first-day return doubled to almost 15% during 1990–1998, before jumping to 65% during the internet bubble years of 1999–2000 and then reverting to 12% during 2001–2003."[50]

These two arguments are easily addressed by looking at the evidence cited herein. It is important to consider the fact that the pre-IPO DLOM studies are the only studies based on transactions in shares of private company stock. Therefore, these studies provide a meaningful starting point in the application of a DLOM for a privately owned ownership interest.

[49]Gregg S. Gaffen, *The Willamette Management Associates Failed IPO Study*, INSIGHTS (Willamette Management Associates), 52-54 (Autumn 2004).

[50]Tim Loughran & Jay Ritter, *Why Has IPO Underpricing Changed Over Time?*, 33 FIN. MGMT. 5, 5 (2004).

C. *The Cost to Obtain Liquidity Studies*

A third type of empirical study is the cost to obtain liquidity study. It is generally accepted that this type of DLOM study only applies to the analysis of a controlling ownership interest. The reason, of course, is that cost to obtain liquidity studies are based on transactions of controlling ownership interests. As a result, the cost to obtain liquidity studies are much less relevant for the analysis of a noncontrolling ownership interest—compared to the analysis of a controlling ownership interest. The application of an illiquidity discount to the analysis of a controlling ownership interest in a privately held company is reasonable. The application of an illiquidity discount to the analysis of a controlling ownership interest has been accepted in numerous judicial decisions. The evidence used to support this valuation adjustment is summarized below.

Costs

No security transfer transaction occurs without costs—both direct costs and indirect costs. The transaction costs that are typically incurred as a result of the sale of a business result from the following:

1. Auditing and accounting fees incurred in preparing financial statements and related information in an understandable and reliable format in order to provide necessary assurances to potential buyers and underwriters.
2. Legal costs incurred during document preparation, the investigation of contingent liabilities, and the negotiation of relevant warranties.
3. Administrative costs (that is, opportunity costs) resulting from the time committed by members of company management in dealing with accountants lawyers, and potential buyers or their representatives—rather than performing their normal, operational duties.
4. Transaction and brokerage costs, if a business broker, investment banker, or other transactional intermediary is involved. These transaction costs are also referred to as "flotation costs"; and when these transaction costs are expressed as a percentage of the sales price, they are referred to as the "gross spread."

The SEC published a study regarding flotation costs in December 1974.[51] The average flotation costs as of that time were approximately 12.4% of the total public offering gross proceeds. In a study published in 1987, Jay R. Ritter conducted a similar analysis of the direct expenses typically incurred by the issuer company in an initial public offering.[52] The results of the Ritter study are summarized in Table 6.

[51]UNITED STATES SECURITIES AND EXCHANGE COMMISSION, COST OF FLOTATION OF REGISTERED ISSUES, 1971-72 (1974).

[52]Jay R. Ritter, *The Costs of Going Public*, 19 J. FIN. ECON. 272 (1987).

Table 6
Ritter Study
Analysis of Flotation Costs

IPO Gross Proceeds[1] ($Million)	Number of Transactions	Underwriting Price Discount[2] (%)	Other Flotation Expenses[3] (%)	Total Cash Expenses (%)
		Firm Commitment IPO Offers		
0.1 – 1.999999	68	9.84	9.64	19.48
2.0 – 3.999999	165	9.83	7.60	17.43
4.0 – 5.999999	133	9.10	5.67	14.77
6.0 – 9.999999	122	8.03	4.31	12.34
10.0 - 120.174175	176	7.24	2.10	9.34
All offers	664	8.67	5.36	14.03
		Best-Efforts IPO Offers		
0.1 – 1.999999	175	10.63	9.52	20.15
2.0 – 3.999999	146	10.00	6.21	16.21
4.0 – 5.999999	23	9.86	3.71	13.57
6.0 – 9.999999	15	9.80	3.42	13.22
10.0 - 120.174175	5	8.03	2.40	10.43
All offers	364	10.26	7.48	17.74

Source: Jay R. Ritter, *The Costs of Going Public*, 19 J. FIN. ECON. 272 (1987).
Notes:
1. Gross proceeds categories are nominal; no price level adjustments have been made.
2. The underwriting discount is the commission paid by the issuing firm; this is listed on the front page of the firm's prospectus.
3. The other expenses figure comprises accountable and nonaccountable fees of the underwriters, cash expenses of the issuing firm for legal, printing, and auditing fees, and other out-of-pocket costs. These other expenses are described in footnotes on the front page of the issuing firm's prospectus. None of the expense categories include the value of warrants granted to the underwriter, a practice that is common with best-efforts offers.

The Ritter study concludes that larger companies generally negotiate lower underwriting fees, as a percent of the IPO gross proceeds. The Ritter study concluded slightly greater average underwriting fees than did the SEC study. More current information is presented in a study conducted by Jay Ritter and Hsuan-Chi Chen published in 2000.[53] In the *Seven Percent Solution*, the authors examined the price spread (that is, the underwriter price discount) from 3,203 firm commitment IPOs from January 1985 to December 1998. The selected IPO transactions all had domestic gross proceeds of at least $20 million before the exercise of the over-allotment option. Table 7 presents the results from this gross price spread study.

Table 7
Ritter and Chen Study
Number of IPOs, Gross Proceeds, and Gross Price Spread Percent

IPO Gross Proceeds:	$20 Million - $80 Million			$80 Million and Up			All IPOs in the Study		
IPO Transaction Date	Below 7%	Exactly 7%	Above 7%	Below 7%	Exactly 7%	Above 7%	Below 7%	Exactly 7%	Above 7%
1985 – 87	46%	26%	28%	76%	12%	12%	52%	23%	25%
1988 – 94	14%	75%	11%	90%	10%	0%	31%	60%	9%
1995 – 98	5%	91%	4%	71%	28%	1%	20%	77%	3%

[53]Hsuan-Chi Chen & Jay R. Ritter, *The Seven Percent Solution*, 55 J. FIN. 1105, 1109 (2000).

Ritter and Chen concluded that a significant number of IPOs were completed with a gross price spread of exactly seven percent. In the 1985 through 1987 period, 23% of all IPOs had a seven percent gross price spread. Of the IPOs studied in 1998 through 1994 period, the amount of transactions with a seven percent price spread increased to 60%. For 1995 through 1998, 77% of all IPOs had a gross price spread of exactly seven percent. In addition, as indicated in the above table, Ritter and Chen observed that the price spread is larger for smaller companies. This evidence indicates that a reasonable underwriter price discount for an IPO is seven percent for companies with IPO gross proceeds exceeding $20 million.

Cost to Obtain Liquidity Study Conclusions

Each of the three cost to obtain liquidity studies presented above concluded that larger companies can negotiate lower underwriter fees, as a percent of the IPO gross proceeds. The most recent Ritter and Chen study presented evidence that reasonable underwriter fees are approximately seven percent of the IPO gross proceeds. However, this study did not analyze companies with IPO gross proceeds of less than $20 million. The SEC study and the Ritter study did analyze companies with IPO gross proceeds under $20 million. They were consistent in finding total measurable costs of over ten percent of the IPO deal size for smaller transactions. Analysts should consider (1) the average results from all cost to obtain liquidity studies, (2) the results by size of IPO transactions from each cost to obtain liquidity study, and (3) other direct and indirect liquidity costs not captured in the above cost to obtain liquidity studies.

The seller of a privately owned company will bear costs in addition to (1) the underwriter fees and (2) the "other costs" estimated above. In the Illiquidity for a Controlling Ownership Interest section, this discussion summarized six factors that contribute to the DLOM for a controlling ownership interest in a privately owned company. These factors relate to (1) uncertain time horizon risk, (2) make ready cost risk, (3) sale price risk, (4) sale proceeds risk, (5) inability to hypothecate risk, and (6) investment banker or other brokerage fees. Only item six, investment banker or other brokerage fees, is included in the seven percent cost to obtain liquidity measured by Ritter and Chen. Accordingly, analysts valuing a controlling ownership interest should consider all potential costs to liquidate controlling equity interests—and not simply apply a DLOM based on the "seven percent solution" or other cost to obtain liquidity studies.

V. DLOM Theoretical Models

There are two types of theoretical DLOM measurement models: (1) option pricing models and (2) discounted cash flow models.

A. *The Option Pricing Models*

Option pricing models assume the cost to purchase stock options relates

directly to the DLOM. Three commonly referenced DLOM studies that rely on option-pricing theory are summarized below.

Chaffe Study

David B.H. Chaffe, III, authored a 1993 DLOM option pricing study in which he related the cost to purchase a (European)[54] put option to the DLOM. Chaffe theorized that, "if one holds restricted or non-marketable stock and purchases an option to sell those shares at the free market price, the holder has, in effect, purchased marketability for those shares. The price of that put is the discount for lack of marketability."[55] Chaffe relied on the Black-Scholes option pricing model to estimate the price of the option in his model. The inputs in the Black-Scholes option pricing model are (1) stock price, (2) strike price, (3) time to expiration, (4) interest rate, and (5) volatility. In the Chaffe model, the stock price and strike price equal the marketable value of the private company stock as of the valuation date; the time to expiration equals the time the securities are expected to remain nonmarketable; the interest rate is the cost of capital; and, volatility is a judgmental factor based on volatility of guideline publicly traded stocks.

According to Chaffe, volatility for small privately owned companies is likely to be 60% or greater. Chaffe reached this conclusion based on the volatility for small public companies that are traded in the over–the-counter market. According to the Chaffe study, the appropriate DLOM for a privately held stock with a two-year required holding period and volatility between 60% and 90% is between 28% and 41%. According to Chaffe, considering that volatility for shares of most smaller,

> privately held companies fit the "VOL 60%-70%-80%-90%" curves, a range of put prices of approximately 28% to 41% of the marketable price is shown at the two-year intercept. At the four-year intercept, these ranges are 32% to 49%, after which time increases do not substantially change the put price.[56]

Chaffe noted that his findings are downward biased (due to the reliance on European options in his model). Therefore, Chaffe concluded that his findings should be viewed as a minimum applicable DLOM.

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[54]"European" options have a single exercise date. In contrast, the holder of an "American" option can exercise the option at any time during the existence of the option.

[55]David B.H. Chaffe, III, *Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations*, 12 BUS. VALUATION REV. 182-86 (1993).

[56]*Id.* at 184. For additional information on using stock options to estimate the DLOM, see Trout, *supra* note 16, the Chaffe study, *supra* note 55, and Longstaff, *infra* note 57. For additional information on stock options, see JOHN C. HULL, OPTIONS, FUTURES AND OTHER DERIVATIVES (6th ed. 2005).

Longstaff Study

Francis A. Longstaff also authored a study that relies on stock options to estimate the DLOM for a private company.[57] Although Chaffe based his study on avoiding losses, Longstaff based his study on unrealized gains. Another difference is that the Longstaff study (allegedly) provides an estimate for the upper limit on the value for marketability. The Longstaff study is based on the price of a hypothetical "lookback"[58] option. The Longstaff study assumes an investor has a single-security portfolio, perfect market timing, and trading restrictions that prevent the security from being sold at the optimal time. The value of marketability, based on these assumptions, is the payoff from an option on the maximum value of the security, where the strike price of the option is stochastic. Table 8 summarizes the results from the Longstaff study.

Table 8
Longstaff Study
Upper Bounds for the DLOM Percentage

Marketability Restriction Period	Standard Deviation = 10%	Standard Deviation = 20%	Standard Deviation = 30%
1 Day	0.421	0.844	1.268
5 Days	0.944	1.894	2.852
10 Days	1.337	2.688	4.052
20 Days	1.894	3.817	5.768
30 Days	2.324	4.691	7.100
60 Days	3.299	6.683	10.153
90 Days	4.052	8.232	12.542
180 Days	5.768	11.793	18.082
1 Year	8.232	16.984	26.276
2 Years	11.793	24.643	38.605
5 Years	19.128	40.979	65.772

As Table 8 illustrates, for a five-year holding period and 30% standard deviation, the appropriate DLOM is over 65%. Longstaff analyzed securities with a volatility between ten percent and 30% because "[t]his range of volatility is consistent with typical stock return volatilities."[59] However, as noted above, small stocks (such as those traded over-the-counter and analyzed by Chaffe) typically have greater volatility, all else equal. According to Longstaff:

> This analysis provides a number of new insights about how marketability restrictions affect security values. First, we show that discounts for lack of marketability can be large even when the length of the marketability restriction is very short. Sec-

[57]Francis A. Longstaff, *How Much Can Marketability Affect Security Values?*, 50 J. FIN. 1767 (1995).

[58]A "lookback" option differs from most other options in that the holder can look back at the end of the option's life and retroactively exercise the option at either the lowest stock price (for a call option) during the holding period or the highest stock price (for a put option) during the holding period.

[59]*See* Longstaff, note 57, at 1771.

> ond, the upper bound provides a benchmark for estimating the valuation effects of marketability restrictions such as circuit breakers, trading halts, and prohibitions on program trading. Finally, these results allow us to assess directly whether empirical estimates of discounts for lack of marketability are consistent with rational market pricing.[60]

Finnerty Study

John D. Finnerty conducted an option-pricing study that, "tests the relative importance of transfer restrictions on the one hand and information and equity ownership concentration effects on the other in explaining private placement discounts."[61] The Finnerty option-pricing study is an extension of the Longstaff study. However, unlike Longstaff, Finnerty did not assume that investors have perfect market timing ability. Instead, Finnerty modeled the DLOM as the value of an average strike put option.

In addition to analyzing stock-options, Finnerty analyzed 101 private placements of restricted stock that occurred between January 1, 1997, and February 3, 1997. The Finnerty private placement study concluded price discounts of 20.13% and 18.41% for the day prior to the private placement and for ten days prior to the private placement, respectively. With regard to his option-pricing model, Finnerty concluded:

> The model (5)-(6) [The Transferability Discount Model] calculates transferability discounts that are consistent with the range of discounts observed empirically in letter-stock private placements for common stocks with volatilities between ☒ = 30% and ☒ = 120% but the implied discounts are greater than (less than) those predicted by the model for lower (higher) volatilities.[62]

In addition, Finnerty made the following observation about the importance of dividends, volatility, and the DLOM:

> My model implies that when the stock price volatility is under 30%, the appropriate discount is smaller than the customary discount range of about 25 to 35%. For example, when ☒ is between 20% and 30% and there is a two-year restriction period, the proper discount is in the range from 15.76% to 20.12% for a non-dividend-paying stock and in the range from 11.50% to 15.96% for a stock yielding 3.0 percent. Also, the halving of the initial restriction period under Rule 144 since February 1997 has roughly halved the appropriate transferability discount.[63]

Finally, Finnerty proposed an explanation for the small price discounts observed in private placement studies conducted by (1) Karen Wruck[64] and

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[60] *Id.* at 1768.

[61] JOHN D. FINNERTY, THE IMPACT OF TRANSFER RESTRICTIONS ON STOCK PRICES at 2 (Oct. 2002), *available at* http://www.bvappraisers.org/contentdocs/Conference/TheImpac_tofTransfer_Restrictions_on_StockPrices.pdf.

[62] *Id.* at 29-30.

[63] *Id.* at 30.

[64] Karen Hopper Wruck, *Equity Ownership Concentration and Firm Value, Evidence from Private Equity Financings*, J. FIN. ECON. 23 (1989).

(2) Hertzel & Smith,[65] where the observed DLOM was less than 15.0%: "The difference is due in part to the information and ownership concentration effects that accompany a common stock private placement, but may also be due to mispricing of the forgone put option. In any case, the discount varies directly with the stock's volatility."[66]

Long-Term Equity Anticipation Securities (LEAPS) Studies

In September 2003, Robert Trout published a Long-Term Equity Anticipation Securities (LEAPS) study.[67] In June 2005, Ronald Seaman updated the Trout LEAPS study.[68] In the winter of 2005, Seaman expanded his earlier study.[69] Each of these LEAPS studies were conducted with similar research logic and with similar research procedures. This discussion concurrently reviews these three LEAPS studies. A LEAP is a long-term put option. LEAPS offer price protection for up to two years in the future. An investor who desires protection against stock price declines can purchase a LEAP put option. The LEAPS studies examined the cost of buying LEAP puts. The cost of the LEAP put option divided by the stock price serves as the basis for the DLOM.

Trout examined nine LEAPS as of March 2003 (with options expiring January 2005). The nine LEAPS examined were for large companies with actively traded securities.[70] According to the Trout study, "The data concerning the relative cost of puts as an insurance premium indicate an insurance premium cost equal to about 24% of price. This finding suggests that the minimum discount that one should assign for the lack of marketability of holding privately held stock is at least 24%."[71]

The Seaman study updated and extended the Trout study up through June, 2005. The Seaman study determined if holding period and risk affected the LEAPS cost (that is, the price discount). This first Seaman study considered 100 randomly selected securities where LEAP options traded. Table 9 summarizes the results of this study.

[65]Michael Hertzel & Richard L. Smith, *Marketability Discounts and Shareholder Gains for Placing Equity Privately*, 48 J. FIN. 459 (1993).

[66]Finnerty, *supra* note 61, at 30.

[67]Robert R. Trout, *Minimum Marketability Discounts*, 22 BUS. VALUATION REV. 124 (2003).

[68]Ronald M. Seaman, *Minimum Marketability Discounts—2nd Edition*, 24 BUS. VALUATION REV. 58 (2005).

[69]Ronald M. Seaman, *A Minimum Marketability Discount*, 24 BUS. VALUATION REV. 177 (2006).

[70]The companies examined include: Amazon, Ford Motor, General Motors, Morgan Stanley, Microsoft, Nextel, Qlogic, Qualcom, and Tyco.

[71]Trout *supra* note 67, at 124-25.

Table 9
Seaman Study Results—Indicated DLOM
Based on Evidence from LEAPS Options

Safety Rank:	1	2	3	4	5
Beta					
Average	0.82	0.95	1.10	1.55	1.87
Median	0.80	0.95	1.10	1.55	1.88
One-Year Price Discount					
Average	7.0%	8.1%	10.6%	15.5%	20.0%
Median	7.3%	7.5%	9.2%	13.8%	17.0%
Two-Year Price Discount					
Average	9.4%	11.1%	14.9%	20.3%	30.3%
Median	9.3%	10.4%	13.8%	18.7%	31.0%

The safety rank measure above is based on *ValueLine Investment Survey*.[72] A measure of 1 implies less risk and a measure of 5 implies greater risk. As demonstrated above, the cost to hedge using LEAP put options increases as the holding period increases. In addition, it is more expensive to hedge as a company's risk increases. Seaman noted that this evidence, "serves primarily as a sanity check on the size of discount for lack of marketability we choose for a particular company."[73] Seaman's second article, also published in 2005, expanded his first article to include the effect size has on the discount. Seaman expanded the number of companies analyzed from 100 to 261. Seaman concluded that, "as company size decreases (in either revenues or assets), the discount required increases."[74]

The authors of the three LEAPS studies concluded that the observed DLOMs are appropriately viewed as benchmark minimum price discounts when applied to privately held companies. The discounts should represent minimum price discounts because: (1) the underlying securities on which the LEAPS are based are often much larger than the privately held subject company, (2) the underlying securities on which the LEAPS are based are marketable, (3) the LEAPS themselves can be sold at any time during the holding period, and (4) there is a known liquidity event (that is, the sale of the underlying security) for the LEAPS

Option Pricing Model Conclusions

The option pricing studies presented above reach price discounts similar to those reached in the empirical studies discussed previously. In the Chaffe, Longstaff, and Finnerty studies, the appropriate DLOM for a privately held company (given certain volatility assumptions) reaches 65%. In the LEAPS studies, the price discount is much lower, but the studies' authors assert that

[72]The ValueLine Investment Survey Online, https://www.ec-server.valueline.com/products/web1.html (last visited Nov. 25, 2007).

[73]Seaman, *supra* note 68, at 59.

[74]Seaman, *supra* note 69, at 177.

the indicated price discount represents a minimum DLOM. Because of their nature, option pricing studies consider only certain aspects of closely held companies. The option pricing studies only consider the factors that affect option pricing the most: holding period and volatility. Although other factors are present in option pricing, the holding period and volatility factors have the greatest impact on option pricing. Therefore, option pricing studies may understate the DLOM because they ignore other factors that may reduce the marketability for a privately held company (for example, contractual transferability restrictions).

Intuitively, basing the size of the DLOM on these two factors makes sense. The holding period is discussed in detail in the restricted stock studies. As the restricted stock studies indicate, the longer the required holding period, the greater price concession (that is, DLOM) a buyer expects to receive. As an example, when the SEC Rule 144 holding period was reduced from two years to one year, the average restricted stock price discount declined. This evidence, combined with the option-pricing studies, indicates that the expected required holding period has a direct impact on the appropriate DLOM for a privately held company. As the expected required holding period increases, so to should the DLOM, holding all other factors constant.

Volatility is directly related to the magnitude of the DLOM. When an investor owns a security that is restricted from trading, the investor assumes the risks (among other risks) of (1) not being able to sell the investment if the value begins to suddenly decline and (2) not being able to sell the investment to reallocate funds to another investment. The first risk factor is materially affected by highly volatile stocks. As volatility increases, the risk of significant stock price depreciation increases. As volatility increases, the risk related to holding a nonmarketable security likewise increases.

The option pricing studies provide a general methodology for analyzing the DLOM. These option pricing studies make several contributions to the empirical research referenced above.

First, the option pricing studies indicate that, for stocks with low volatility, the appropriate DLOM may be below the range of average price discounts reported in the empirical studies. The practical problem, of course, is how to determine the volatility for the stock of a privately owned company. The Chaffe study assumed that the appropriate volatility for most privately held companies is likely in excess of 60%, based on the average volatility of small publicly traded company stocks. In contrast, Longstaff analyzed price discounts for companies with volatility ranging between ten percent and 30%, based on publicly traded companies of all sizes.

The analyst using these data must consider whether the subject company qualitative and quantitative factors warrant a low estimated volatility. If so, these factors may support the use of a lower than average DLOM. Alternatively, if the subject company would warrant a volatility of 60% or greater, then the appropriate DLOM may be above the average DLOM indicated in the various DLOM studies (all other factors being equal).

Second, the option pricing studies show how the length of the required holding period has a material effect on the magnitude of the DLOM. As the Longstaff study concluded, as the holding period increases from 1 year, to two years, to five years, the DLOM for a stock with a 20% volatility increases from 17%, to 25%, to 41%. The Chaffe and Finnerty studies reached similar conclusions regarding the required holding period and the DLOM. If, due to contractual restrictions, a limited pool of potential buyers, or other factors, the subject company stock is not expected to become marketable for many years, then the use of a greater than average discount may be appropriate. However, if the subject company is contemplating a liquidity event such as IPO, merger, or sale, then the DLOM may be lower than the average price discount indicated by the empirical studies.

The usefulness of these option pricing studies is limited by at least two factors. First, the securities that are the subject of the option pricing studies (and the options themselves) are liquid. In contrast, the lack of a trading market and the presence of transfer restrictions cause private company stock to be relatively illiquid. This difference between the option pricing studies and private company stock would warrant a price discount greater than what is indicated by the option pricing studies. Second, an owner of private company stock does not have the ability to hedge his or her investment in the options market. Stock options on small, thinly traded companies rarely exist, and the market for private company stock or options on that stock simply does not exist. If the implied DLOM from a particular option pricing model is 30% (when the strategy is actually available to investors), then the implied DLOM for shares of private company stock would be even greater (when the strategy does not actually exist).

B. *The Discounted Cash Flow Models*

A second category of theoretical studies is based on the discounted cash flow method. The discounted cash flow method is based on the principle that value equals the present value of future income. Z. Christopher Mercer and Travis W. Harms described how the discounted cash flow model relates to the DLOM:

> Quantitative analyses therefore estimates the value of illiquid interests based on the expectation of benefits (distributions or dividends and proceeds of ultimate sales) over relevant expected holding periods using appropriate discount rates to equate with present values. The process of doing this analysis, in the context of valuing a business at the marketable minority interest level, determines the applicable marketability discount.[75]

Two studies that rely on the discounted cash flow methodology are summarized below.

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[75]Z. Christopher Mercer & Travis W. Harms, *Marketability Discount Analysis at a Fork in the Road*, 20 BUS. VALUATION REV. 21, 23 (2001).

The Quantitative Marketability Discount Model (QMDM).

The Quantitative Marketability Discount Model (QMDM) is a shareholder-level discounted cash flow model that uses a quantitative analysis to calculate the DLOM precisely. Developed by Z. Christopher Mercer, the QMDM calculates the DLOM based on (1) the expected growth rate in the subject company value, (2) the expected interim cash flow, (3) the expected holding period, and (4) the required holding period return. In his book, *Quantifying Marketability Discounts*, Mercer provides detailed guidance with regard to estimating these four factors.[76]

In the application of the QMDM, an analyst first values the subject company at the entity level, resulting in a stock valuation as if the stock was readily marketable. Next, the shareholder value is calculated. The shareholder value represents the nonmarketable value of the subject stock. To calculate the shareholder value, the analyst increases the value of the subject company by the growth rate during the expected holding period. The analyst next discounts the future company value using the required holding period return. The analyst then adds the present value of the dividend stream received during the holding period to this present value. The resulting value equals the shareholder value. The calculation of one minus the ratio of shareholder value to enterprise value equals the DLOM, based on the QMDM.

While intuitively appealing, the QMDM has practical limitations. For example, the DLOM computed using the model is highly subject to the model inputs. In *Estate of Weinberg v. Commissioner*,[77] the Tax Court noted that "slight variations in the assumptions used in the model produce dramatic differences in the results."[78] In *Janda v. Commissioner*,[79] the Tax Court was concerned with the magnitude of the DLOM calculated using the QMDM model: "We have grave doubts about the reliability of the QMDM model to produce reasonable discounts, given the generated discount of over 65%."[80]

Tabak Model

The Tabak model is another discounted cash flow model used to estimate the appropriate DLOM for a privately owned company based on the capital asset pricing model (CAPM).[81] The Tabak model presents a unique way to estimate the DLOM. This is because the Tabak model

> focuses on the extra risks imposed on the owner of a security or interest in a business enterprise, and not on the lack of access to capital. In brief, the theory uses market data on the additional return that investors require in order to hold a risky asset,

[76]Z. Christopher Mercer, Quantifying Marketability Discounts (1997).
[77]79 T.C.M. (CCH) 1507, 2000 T.C.M. (RIA) ¶ 2000-051.
[78]*Id.* at 1515, 2000 T.C.M. (RIA) ¶ 2000-051 at 292.
[79]81 T.C.M. (CCH) 1100, 2001 T.C.M. (RIA) ¶ 2000-024.
[80]*Id.* at 1104, 2001 T.C.M. (RIA) ¶ 2000-024 at 195.
[81]David Tabak, A CAPM-Based Approach to Calculating Illiquidity Discounts (2002), *available at* http://www.nera.com/image/5657.pdf.

measured by the equity risk premium, to extrapolate the extra return that the holder of an illiquid asset would require.[82]

Discounted Cash Flow Model Conclusions

The discounted cash flow models provide an interesting analysis regarding (1) the cause and (2) the measurement of the DLOM. Although the discounted cash flow models are controversial, they are based on generally accepted financial theory. For example, calculating the present value of the cash flow received at the "shareholder level" during the expected holding period, as in the QMDM, is a theoretically correct process. However, the model results are highly sensitive to the model inputs. In addition, the model inputs used in the QMDM and the Tabak model require the application of analyst's judgment—that is, a subjective factor that the models attempt to overcome. Finally, the QMDM, and especially the Tabak model, have not been widely accepted by valuation analysts or by the federal courts. Because of these factors, valuation analysts should only rely on the discounted cash flow models presented above when the inputs can be measured with relative certainty. In addition, an analysis of the empirical DLOM models should typically accompany the use of the discounted cash flow DLOM models.

VI. Consideration of Specific Transferability Restrictions

The restricted stock DLOM studies summarized above present a multitude of factors that affect the DLOM for privately owned companies. Certain factors purported to affect the DLOM appear more often than others. For example, many of the restricted stock studies agree that company size, block size, and dividends all have an effect on the DLOM. However, these DLOM studies are limited because they can only consider the factors that they can measure. There are specific factors that affect privately owned companies that are not present (and therefore not measurable) in the various restricted stock studies. These factors are contractual restrictions, such as a shareholder agreement, right of first refusal, buy-sell agreement, and the like.

These contractual restrictions can severely limit the marketability of a noncontrolling ownership interest in a privately owned company. These factors, by their nature, are company-specific. The following list presents the contractual restrictions that may affect the size the DLOM:

1. Buy-sell agreements;
2. Shareholder or partnership agreements;
3. Rights of first refusal; and
4. Other contractual transferability restrictions.

These contractual restrictions can severely limit the marketability of the privately owned equity interest. The more restrictive the agreement or provision, the greater the appropriate DLOM, all else equal

[82] *Id.* at 10.

VII. Factors Affecting the Selection of the DLOM

As stated previously, an asset is not simply either marketable or nonmarketable. Rather, there are varying degrees of marketability. The studies discussed above describe a starting point to estimate the DLOM. However, the facts and circumstances of each analysis will determine the appropriate DLOM. It is a matter of analyst judgment to select a DLOM based on (1) the empirical DLOM evidence, (2) the theoretical DLOM evidence, and (3) the facts and circumstances of each individual case. This section discusses the case-specific factors that affect the analyst's selection of the DLOM.

In *Mandelbaum v. Commissioner*,[83] the court cited nine specific (but nonexclusive) factors for analysts to consider in developing a DLOM (or discount for lack of liquidity):

1. Financial statement analysis;
2. Dividend history and policy;
3. Nature of the company, its history, its position in the industry, and its economic outlook;
4. The company management;
5. The amount of control in the transferred shares;
6. The restrictions on transferability;
7. The holding period for the stock;
8. Subject company's redemption policy; and
9. Costs associated with a public offering.

Mandelbaum has been cited frequently in subsequent decisions related to the measurement of the DLOM. The *Mandelbaum* factors are intuitive, and they reconcile with the empirical studies discussed above. Analyses of the *Mandelbaum* factors, the empirical studies, the theoretical studies, and other DLOM literature make it clear that dozens of company-specific and security-specific factors affect the magnitude of the DLOM. These dozens of factors generally fall into three categories: (1) dividend payments, (2) expected holding period, and (3) subject company risk. A discussion of these three categories of DLOM factors follows.

A. *Dividend Payments*

The text *Valuing a Business*[84] clearly explains the importance of dividends:

> Stocks with no or low dividends suffer more from lack of marketability than stocks with high dividends. Besides being empirically demonstrable, this makes common sense. If the stock pays no dividend, the holder is dependent *entirely* on some future ability to sell the stock to realize any return. The higher the dividend, the greater the return the holder realizes without regard for sale of the stock.[85]

[83] 69 T.C.M. (CCH) 2852, 2862-62, 1995 T.C.M. (RIA) ¶ 95,255 at 1614.
[84] PRATT ET AL., *supra* note 47.
[85] *Id.* at 417.

As this quote illustrates, an investor in a privately owned company would generally prefer some dividends to no dividends. However, when the subject ownership interest is a noncontrolling ownership interest, the analyst should also consider that the level of future dividends may not equal the level of historical dividends. For example, assume a closely held company makes an annual dividend payment equal to 100% of its annual cash flow. Furthermore, assume that all company shareholders are related. Under a typical interpretation of the fair market value standard of value, which disregards family attribution, the willing buyer of a noncontrolling interest in the equity of this hypothetical company will not be a family member. In order for the economic benefits to remain within the family, the subject company might (1) cease paying dividends and (2) allocate the capital previously used for dividends to the family members in the form of increased salaries, for instance. In this example, the presence of historical dividends is not the only factor to consider when analyzing dividends relative to a private company. The expected future dividends of the subject company should typically be considered in a DLOM analysis. This example also illustrates the importance of considering the facts and circumstances of each case—and of not simply applying a DLOM based on average DLOM percentages.

B. *Expected Holding Period*

The second factor that affects the selection of the DLOM is the expected holding period. *Mandelbaum* specifically identified the expected holding period as a factor that affects the DLOM. Similarly, Revenue Ruling 1977-287[86] expressly states that the expected holding period affects the DLOM. The restricted stock studies, the pre-IPO studies, the option-pricing studies, and the discounted cash flow models all consider holding period as a factor. This factor is frequently associated with the DLOM because (1) it has been clearly measured in empirical studies, (2) it is intuitive, and (3) it encompasses a variety of other intuitive factors.

In Table 10, the size of the DLOM is clearly related the expected holding period. As the holding period increases, so does the DLOM. The expected holding period for a privately held company is clearly an important consideration in the selection of the DLOM. One challenge for analysts is to assess accurately the expected holding period of privately owned stock. Analysts can estimate the expected holding period based on consideration of (1) put rights, (2) the prospect of an IPO or sale of the business, (3) the size of the subject interest, and (4) contract-specific transferability restrictions.

[86]Rev. Rul. 1977-287, 1977-2 C.B. 319 at § 6.02 ("[T]he longer the buyer of the shares must wait to liquidate the shares, the greater the discount.").

Table 10
Emory Studies for 1980 to 2000 (after a 2002 Revision)
Price Discounts Versus Time Between Transaction & IPO

Days	Average	Median	Count
0-30	30%	25%	18
31-60	40%	38%	72
61-90	42%	43%	162
61-120	49%	50%	161
121-153	55%	54%	130
TOTAL			543

Source: Institute of Business Appraisers Annual National Conference, June 2, 2003.

Put Rights

In some situations, company shareholders will posses certain "put" rights. These put rights allow the shareholders to resell their shares to the company based on certain specified provisions. For example, in an employee stock ownership plan (ESOP), the put right may allow all of the ESOP participants to resell their shares to the company upon the termination, retirement, death, or disability of the participant, at the then fair market value. The existence of a put option provides a market for shares of privately owned stock and would decrease the appropriate DLOM, all else equal.

The existence of a put option has little value if the subject company is not expected to have the ability to meet its stock redemption obligation. When a privately held company has a put option, the ability of the company to acquire shares that are "put" to the company is typically an important factor to consider. The put option may not actually provide liquidity to a shareholder in a company that generates negative operating cash flow, or that has little debt capacity.

Prospect of a Public Offering or a Sale of the Business

Another obvious liquidity event for a noncontrolling shareholder in a privately owned company is an IPO or a sale of the business. Here, the greater the imminence of an IPO or sale, the lower the appropriate DLOM, all other factors being equal.

Among other factors, the imminence of an IPO or sale depends on the size of the business, the industry the business competes in, and the attitudes of the company owners. A likely IPO/sale candidate may be characterized by the following factors: (1) the company maintains a strong balance sheet, (2) the company enjoys consistent and projected profitability, (3) the company operates in an industry under consolidation, and (4) the company is actively marketed with the assistance of a business broker.

Of course, like marketability itself, this factor is not an absolute proposition. That is, a company should not be characterized as "likely" or "unlikely" to complete an IPO or sale. Instead, there is a spectrum of liquidity event possibilities that exist. Even though a company may be a strong candidate for a liquidity event transaction, a noncontrolling equity owner has no assurance (and no influence) that this type of transaction will occur. The potential for a liquidity event, and the risks of completing a liquidity event both affect the

expected holding period of a stock.

The Size of the Interest

The size of the privately owned interest will also affect the expected holding period. There is empirical evidence that larger blocks of stock are associated with a larger DLOM. Larger blocks of stock are typically harder to sell. The pool of potential buyers that can afford an asset decreases as the price of the asset increases. That is, more investors can afford (in terms of price, and in the context of their investment objectives) to acquire a $100,000 interest than a $10 million interest.

Contractual Transferability Restrictions

Contractual transferability restrictions include buy-sell agreements, shareholder agreements, and the like. In many privately owned holding companies, these agreements are put in place to ensure company ownership is kept within the family. Therefore, these agreements often severely restrict the transferability of the subject stock. These agreements are not present only in family-owned holding companies. They appear in S corporations, C corporations, and virtually every other business entity available. Any agreement that restricts the transfer of stock tends to increase the amount of the DLOM.

C. *Subject Company Risk*

The third factor that affects the DLOM is the subject company risk. The restricted stock studies and the option pricing studies both conclude that the size of the DLOM is directly related to the stock price volatility (one measure for risk). The studies are also consistent in attributing company size (another measure for risk) with the DLOM size. For example, the McConaughy, Cary and Chen restricted stock study pointed out, "There are three factors that remain significant: size, stability of revenue growth, and stock price volatility. These three factors clearly reflect the riskiness of investing in a company."[87] Each of these three factors directly relates to the subject company risk.

Analysts agree that a large company is a "safer" investment than a similar small company, all other factors being equal. This conclusion is illustrated by comparing the required rates of returns on large-capitalization companies to small-capitalization companies. Ibbotson Associates makes this comparison in *SBBI Valuation Edition 2006 Yearbook (SBBI)*:

> One of the most remarkable discoveries of modern finance is that of a relationship between firm size and return. The relationship cuts across the entire size spectrum but is most evident among smaller companies, which have returns on average than larger ones . . . . Small-capitalization stocks are still considered riskier investments than large company stocks. Investors require an additional reward, in the form of additional return, to take on the added risk of an investment in small-capitalization stock.[88]

[87]McConaughy et al., *supra* note 31, at 46.

[88]Ibbotson Associates, SBBI Valuation Edition 2006 Yearbook: Valuation Edition 129, 160 (2006).

Large companies are perceived as safer investments than are small companies. This is because greater earnings typically enable a company (1) to withstand downswings in the economy and subject company industry and (2) to capitalize on growth opportunities that require borrowing capacity or cash.

Factors in addition to size can also affect subject company risk. Every company is unique, and there is no one list that can encompass the wide range of potential risk factors. However, the following list includes some of the common company-specific factors that may affect subject company risk:

1. Historical financial ratios;
2. Historical earnings, trends, and volatility;
3. Management depth;
4. Product line diversification;
5. Geographic diversification;
6. Market share;
7. Supplier dependence;
8. Customer dependence;
9. Deferred expenditures; and
10. Lack of access to capital markets.

VIII. Current Controversies Regarding DLOM Analyses

The methods used to estimate the appropriate DLOM are still evolving. The proof of this statement is the frequency of newly published empirical and theoretical studies that discuss the DLOM. Thus, analysts should keep abreast of the latest developments and controversies with regard to the DLOM. This section presents some of the current DLOM controversies.

A. *Use of Multiple Regression Analysis to Estimate DLOM*

While *Mandelbaum* and certain restricted stock studies provide analysts with guidance regarding the DLOM, neither *Mandelbaum* nor the studies were able to translate company-specific factors into a point estimate for the DLOM. It is important for the analyst to consider the case-specific factors and the nature of the restricted stock studies. It is equally important for the analyst to understand how these factors affect the DLOM. One way to reach this understanding is to use a multi-factor formula (that is, a regression analysis) that may estimate the DLOM with precision.

Certain of the empirical studies (for example, the Silber, Trout, and Hertzel & Smith studies) attempted to provide precise estimates of the DLOM using a formula. These models have an initial appeal because they (1) estimate a precise DLOM for a closely held company and (2) are based on empirical data. However, formula-based methodologies are problematic.

The most significant problem with regression-based DLOM models is the fact that the formulas only consider a limited number of variables. For example, the Silber model considers four variables: (1) revenue, (2) earnings, (3) size of block, and (4) relationship between buyer and seller. Many relevant

valuation variables are omitted from the Silber formula. Silber does not consider (1) the expected holding period for the subject securities, (2) the level of subject company distributions, or (3) any transfer restrictions that may affect the subject stock. These three factors—which are not considered by Silber—are three of the most important factors that affect the magnitude of the DLOM. The other regression-based DLOM studies suffer from this same variable constraint problem.

In a simple example, John Kania,[89] an analyst employed by the Service, assumed a closely held company (1) has $60 million in annual revenue, (2) positive earnings, (3) a share block equal to six percent, and (4) no special relationship between the selling company and investor. Using these assumptions, Kania calculated a DLOM of 19% from the Silber model. However, there are significant other factors that could influence the DLOM. For example, a shareholder agreement could severely restrict the ability to transfer the subject shares. This would cause the DLOM to increase, all else equal. Alternatively, the subject company could be in the process of selling its common equity. This would cause the DLOM to decrease, all else equal.

A second potential problem with regression-based models is the potential for forecast errors. Dr. Stanley Jay Feldman, a professor and valuation analyst, warns:

> [Regression-based] models should not be used for [selecting a DLOM] because the forecast errors are likely to be large. Moreover, based on the structure of these models and their prediction errors, it is not possible to state with any certainly that a 13.5% discount is statistically different than a discount of say 25%.[90]

Even after applying a regression model, analysts should still consider the qualitative and quantitative factors that affect the DLOM.

B. *Disaggregation of DLOM Attributions*

The Hertzel & Smith study and the Bajaj study both disaggregated the observed private placement price discount into (1) lack of liquidity and (2) other factors. These studies accomplished this disaggregation using statistical analysis. This disaggregation of DLOM attributes has stimulated considerable debate in the valuation community. This section summarizes the debate.

Hertzel & Smith analyzed the difference in price discounts between private placements of restricted shares and unrestricted shares. According to Hertzel & Smith, "We find an additional discount of 13.5% for placements of *Restricted shares.*"[91] The 13.5 price discount concluded in the Hertzel & Smith private placement study is often cited in Tax Court judicial decisions and in the valuation literature. However, there are at least three reasons why

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[89]John J. Kania, *Predicting Lack of Marketability Discounts by Use of an Economic (Statistical Regression) Model*, 21 BUS. VALUATION REV. 178 (2002).

[90]Stanley Jay Feldman, *A Note on Using Regression Models to Predict the Marketability Discount*, 21 BUS. VALUATION REV. 145 (2002).

[91]Hertzel & Smith, *supra* note 21, at 480.

valuation analysts should be cautious when relying on the Hertzel & Smith price discount conclusion.

First, the purpose of the Hertzel & Smith study was not to determine the DLOM. Rather, Hertzel & Smith were generally examining private placement price discounts.

Second, the number of private placements involving restricted stock was small—both (1) in absolute terms (there were only 18 such private placements) and (2) relative to the number of private placements of unrestricted stock (there were 88 of such placements). Without additional data on the 18 restricted private placements, the Hertzel & Smith results require additional testing to verify the accuracy and the relevance of the study.

Third, the average placement size was 15.98% of the outstanding stock. When commenting on the observation of private placements completed at a price premium relative to the share price ten-days prior to the announced placement, Hertzel & Smith recognized that "[p]remiums appear to reflect the value of control, cash infusions by investors who already own much of the outstanding stock, and market price declines between the time the placement price is negotiated and when it is announced to the market."[92] It is impossible to know (based on the amount of data that Hertzel & Smith made available) whether or not any of these effects are present in the 18 restricted private placements used to conclude the 13.5% price discount. If so, the lower than expected price discount would likely be mitigated by a price premium for ownership control, even if such ownership control was not absolute.

Like Hertzel & Smith, the authors of the Bajaj study hypothesized that the observed private placement price discounts were due to factors other than illiquidity. Specifically, they attributed the observed price discounts to the following four factors: (1) the fraction of total shares offered in the placement; (2) business risk, as measured by volatility in the issuer's publicly traded shares; (3) financial distress, as measured by Altman's Z-score; and (4) the total proceeds from the private placement. According to the Bajaj study, "controlling for all other factors influencing private placement discounts, an issuer would have to concede an additional discount of 7.23% simply to compensate the buyer for lack of marketability."[93]

The Bajaj study is conceptually related to the Hertzel & Smith study. This is because both studies (1) compare registered private placements with unregistered private placements and (2) determine an average price discount. The Bajaj study, however, addressed two of the three potential problems present in the Hertzel & Smith study. First, estimating the DLOM was not the purpose of the Hertzel & Smith study. Estimating the DLOM is exactly what the Bajaj study attempted to do. Second, the Bajaj study analyzed 50 or 51

[92] *Id.* at 470.

[93] Bajaj et al., *supra* note 26 at 114.

unregistered private placements (depending on whether Bajaj's Table 5 is correct or the text that accompanies Table 5 is correct). Alternatively, there were only 18 unregistered private placements in the Hertzel & Smith study.

The Bajaj study attempted to improve the Hertzel & Smith study by explaining the specific causes for the observed price discount. As indicated above, Bajaj concluded the difference between registered issues and unregistered issues was 22.2%. However, the Bajaj study attributed only 7.23% of that price discount to illiquidity. The remaining price discount was attributed to the fact that the unregistered placements (1) are made by financially weaker firms and (2) involve smaller private placement proceeds—both factors that warrant a greater DLOM, all else equal.

In spite of these methodological improvements, the Bajaj study (as well as the Hertzel & Smith study) may be downwardly biased by the presence of an ownership control price premium. The average ratio of shares offered to total shares in the Bajaj study equaled 15.87%. Publicly traded stocks have relatively diffuse ownership. Therefore, a 16% ownership interest represents a significant ownership interest in the equity of a publicly traded company. If the private placement price was upward influenced because the purchaser was receiving elements of ownership control (for example, the ability to elect a member of the board of directors or influence management decisions), then the DLOM calculated in the Bajaj study would be understated.

When relying on the Bajaj study results to estimate the DLOM for a privately owned company, it is important for the analyst to understand the difference between the subject private company and the companies included in the Bajaj study. Many privately owned companies are more comparable to the companies issuing unregistered private placements than to companies issuing registered private placements. In fact, it is likely that the subject private company is considerably more risky than either of those two groups of comparables. When compared to the unregistered private placements in the Bajaj study, many private companies have greater financial risk and a much longer expected investment holding period. For these reasons, it may be appropriate to consider the Bajaj study total 22.2% price discount as a lower bound for the DLOM and not, as the authors purport, to consider the 7.23% price discount as the relevant DLOM for privately owned companies.

IX. Summary and Conclusion

A. *Review of Lack of Marketability Issues*

The DLOM adjustment is often a controversial issue in a business or security valuation. Analysts, courts, and educators have all opined as to the correct interpretation of the DLOM. There are dozens of studies that attempt to quantify the DLOM. Nonetheless, the current controversies still include (1) the relevance of certain DLOM studies, (2) the interpretation of certain DLOM studies, (3) the proper methodology to measure the DLOM for a privately owned company, and (4) the appropriate size of the DLOM

adjustment.

In spite of the disagreements regarding the DLOM, a body of literature supports the existence of the DLOM. Restricted stock studies, the first type of analysis performed to estimate the DLOM empirically, first appeared in 1971. Between the 1971 SEC restricted stock and today, the DLOM literature has developed considerably. Studies have expanded to include pre-IPO DLOM studies, option pricing studies, and discounted cash flow models. The studies have affirmed that smaller, riskier companies warrant a greater DLOM, all other factors being equal. The studies are also consistent in attributing a larger DLOM (1) to companies that do not pay dividends and (2) to ownership interests with longer expected investment holding periods. Between 1971 and 2007, dozens of DLOM studies were published that affirm what the 1971 SEC study concluded: (1) the DLOM is an economically valid concept; and (2) the size of the DLOM is often substantial.

B. *Application of DLOM in the Valuation Process*

In estimating the appropriate DLOM, an analyst should consider all of the facts and circumstances relevant to the subject security or business interest. Based on the unique facts of a specific analysis, there are times when one study is more relevant than another study. Marketability and lack of marketability are relative (and not absolute) terms.

The restricted stock studies conducted prior to 1990 indicated price discounts of around 35%. After 1990, the DLOM indicated in the restricted stock studies decreased to around 25%. This decrease was due to (1) the improved liquidity in the market for restricted stocks and (2) a shorter required holding period based on SEC Rule 144. The average DLOM indicated in the pre-IPO studies was approximately 45% to 50%. Finally, the "cost to obtain liquidity studies" concluded that (1) underwriter fees are approximately seven percent and (2) other measurable transaction-related fees range from five percent to seven percent. The difference in the degree of marketability in the subject ownership interests analyzed in the various DLOM studies is the primary cause for the different average price discounts observed in the studies.

Analysts should not naively select a DLOM based solely on the average price discount reported in the empirical studies. A thorough understanding of how the subject interest compares to the interests analyzed in the various empirical DLOM studies is an important component of defensible DLOM analysis. If the subject company or subject security has an expected holding period of one year or less, then it may be more meaningful to place more emphasis on the DLOM results from the post-1990 restricted stock studies than the pre-IPO studies. If a public market or liquidity event is not expected to occur for many years, then the results from pre-IPO DLOM studies may be more meaningful.

In addition to comparing the subject interest to the empirical DLOM studies, the analyst should consider whether the subject interest warrants an upward or downward adjustment relative to the selected benchmark.

As mentioned throughout this Article, the most important company-specific and security-specific factors to consider are: (1) dividend payments, (2) expected holding period, and (3) subject company risk. The importance of analyzing the subject company relative to the empirical DLOM studies is illustrated by the wide range of discounts observed within each empirical DLOM study. The wide range of observed discounts illustrates that a multitude of company-specific and security-specific factors affect stock pricing and the size of the DLOM.

The analyst should only rely on the "cost to obtain liquidity studies" when the subject security is sufficiently similar to the securities analyzed in these studies. Because these studies relate to selling a controlling ownership interest, analysts typically consider these data when selecting a lack of liquidity for a controlling ownership interest. The primary relevance of these studies for noncontrolling ownership interests is the extent to which they support the significant cost (both measurable and not measurable) that goes into selling a privately owned company.

For the reasons mentioned throughout this Article, the theoretical studies summarized above have limited practical use. These studies are problematic because of (1) the limited number of factors considered, (2) the lack of acceptance in the courts and the professional valuation community, and (3) the quantitative sensitivity of the model inputs.

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