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Value & Cents

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Methodologies for Arriving at DLOM

Valuation analysts often value closely held company securities for various bankruptcy purposes. Depending on the valuation approaches and methods applied, and the benchmark empirical data being used, these analyses may initially conclude the security value on a marketable (as if freely traded on a stock exchange) basis. In such instances, analysts might have to apply a discount for lack of marketability (DLOM) adjustment in order to arrive at the final value.

This article summarizes the factors to consider to measure the DLOM associated with noncontrolling securities of a closely held company. This security-level DLOM is different from the entity-level DLOM that is applied at the closely held company level.

DLOM Analytical Models

There are two types of models to measure the security level DLOM: empirical and theoretical. Empirical models are based on capital market transaction observations. There are two categories of empirical DLOM studies: (1) price discounts on public company restricted stock sales (restricted stock studies); and (2) price discounts on private stock sales prior to an initial public offering (pre-IPO studies). Theoretical models do not directly measure the DLOM from transactional data. There are two categories of theoretical models: option-pricing models (OPMs) and discounted cash flow (DCF) models.

Empirical Models Restricted Stock Studies

Public companies sometimes raise capital by completing a private placement of debt or equity securities. In an equity private placement, a company can issue either registered or unregistered (*i.e.*, restricted) stock. Registered stock can be traded on the stock exchange, but unregistered stock can-

not be registered for trading on a stock exchange. Unregistered stock is typically sold at a discount compared to the price of the (registered) publicly traded stock.

Public companies might be willing to accept this price discount because the time and cost of

Exhibit 1: Restricted Stock Studies Implied DLOM

Restricted Stock Study	Observation Period of Study	Observed Average or Median Price Discount
SEC Overall Average	1966-69	25.8%
SEC Nonreporting OTC Companies	1966-69	32.6%
Milton Gelman	1968-70	33.0%
Robert R. Trout	1968-72	33.5%
Robert E. Moroney	1969-72	35.6%
J. Michael Maher	1969-73	35.4%
Standard Research Consultants	1978-82	45.0%
Willamette Management Associates	1981-84	31.2%
Hertz and Smith	1980-87	20.1%
William L. Silber	1981-88	33.8%
Bajaj, Denis, Ferris and Sarin	1990-95	22.2%
Johnson Study	1991-95	20.0%
Management Planning Inc.	1980-96	27.0%
FMV Opinions Inc.	1980-2014	19.3%
Greene and Murray	1980-2012	24.9%
Columbia Financial Advisors Inc.	1996-97	21.0%
Columbia Financial Advisors Inc.	1997-98	13.0%
LiquiStat	2005-06	32.8%
Angrist, Curtis and Kerrigan	1980-2009	15.9%
Stout Risius Ross	2005-10	10.9%



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registering the new stock with the Securities and Exchange Commission (SEC) would make the stock issuance impractical. These observed price discounts (*i.e.*, public stock price compared to same company private stock price) indicate a DLOM.

SEC Rule 144 governs the purchase and sale of stock issued in unregistered private placements. Until 1995, restricted stock sale transactions were reported to the SEC. Since 1995, analysts have collected restricted stock sale transaction data from private sources.

Restricted Stock Study Conclusions

Exhibit 1 summarizes 20 restricted stock studies spanning the late 1960s through 2013. The decrease in price

discounts over time may be due to the following factors: (1) the increase in volume of privately placed stock under SEC Rule 144(a); and (2) the change in the minimum SEC-required holding period under Rule 144 — from two years to one year — that took place as of April 29, 1997.

Pre-IPO Studies

A pre-IPO study examines security sales of a closely held company that subsequently went public. The DLOM is quantified by the difference between (1) the public market price of the IPO and (2) the private transaction price at which a stock was sold prior to the IPO.

Emory Studies: Between 1980 and 2000, a number of studies were conducted under the direction of John Emory.

Exhibit 2: Emory Pre-IPO Implied DLOM

Pre-IPO Study	Number of Prospectuses Reviewed	Number of Qualifying Transactions	Indicated Price Discount	
			Mean	Median
1980-81	97	12	59%	68%
1985-86	130	19	43%	43%
1987-89	98	21	38%	43%
1989-90	157	17	46%	40%
1990-91	266	30	34%	33%
1992-93	443	49	45%	43%
1994-95	318	45	45%	47%
1995-97	732	84	43%	41%
1997-2000	1,847	266	50%	52%

Exhibit 3: Valuation Advisors Pre-IPO Study Implied Median DLOM

IPO Year	Period Before IPO in Which Transaction Occurred					Number of Transactions
	0-3 Months	4-6 Months	7-9 Months	10-12 Months	1-2 Years	
1995	37.82%	28.62%	60.40%	50.33%	60.64%	34
1996	30.83%	52.97%	56.37%	69.38%	71.81%	270
1997	34.18%	50.00%	67.12%	76.01%	80.00%	212
1998	23.35%	46.67%	68.93%	71.41%	71.91%	212
1999	30.77%	53.89%	75.00%	76.92%	82.00%	694
2000	28.70%	45.08%	61.51%	68.92%	76.64%	653
2001	14.74%	33.17%	33.38%	52.06%	51.61%	115
2002	6.15%	17.33%	21.88%	39.51%	55.00%	81
2003	28.77%	22.30%	38.36%	39.71%	61.37%	123
2004	16.67%	22.68%	40.00%	56.25%	57.86%	334
2005	14.75%	26.10%	41.68%	46.11%	45.45%	296
2006	23.47%	20.69%	40.23%	46.51%	56.27%	264
2007	12.67%	32.55%	43.69%	56.00%	54.17%	459
2008	20.00%	24.21%	45.85%	52.17%	41.18%	41
2009	6.16%	31.85%	26.82%	41.00%	34.87%	108
2010	15.81%	29.89%	44.42%	47.54%	51.88%	358
2011	23.27%	34.62%	43.26%	50.78%	62.10%	281
2012	18.86%	24.07%	28.90%	35.48%	44.78%	292
1995-2012 Average	21.50%	33.15%	46.54%	54.23%	58.86%	—
2008-2012 Average	16.82%	28.93%	37.85%	45.39%	46.96%	—

The observations in each study consisted of companies with an IPO in which Emory's firm either participated or received a prospectus. The prospectus for each of the 4,088 offerings was analyzed to determine the relationship between the IPO price and the price at which the latest private transaction took place (up to five months prior to the IPO). The mean and median price discounts from all of the transactions analyzed equal 46 percent and 47 percent, respectively. Exhibit 2 summarizes the results of the Emory studies.

Valuation Advisors LLC Studies: Valuation Advisors created a database that includes more than 3,500 pre-IPO transactions that occurred within two years of an IPO. These transactions are arranged into five time periods: four three-month intervals for the 12 months immediately before the IPO, and a single period for the time frame from one to two years before the IPO. Exhibit 3 summarizes the results of these studies.

Willamette Management Associates (WMA) Studies: WMA prepared 18 pre-IPO studies covering a period from 1975 through 2002. Due to the small sample size, the data from 2001 and 2002 were excluded from the analysis. The results of the WMA studies are summarized in Exhibit 4.

The WMA pre-IPO average price discounts were generally greater than the restricted stock average price discounts. One explanation is the fact that — unlike pre-IPO transactions — restricted stock transactions involve companies that already have an established public-trading market.

Theoretical Models

OPMs

OPMs are based on the premise that the cost to purchase a stock option is related to the DLOM. The following discussions summarize four DLOM studies that rely on OPMs.

Chaffe Study: David Chaffe authored a study in which he related the cost to purchase a European put option to the DLOM. He relied on the Black-Scholes OPM to estimate the option price. The inputs in the Black-Scholes model are as follows: (1) stock price; (2) strike price; (3) time to expiration; (4) interest rate; and (5) volatility.

In the Chaffe model, the stock and strike prices equal the marketable value of the closely held company stock as of the valuation date, the time to expiration equals the time the securities are expected to remain nonmarketable, the interest rate is the cost of capital, and volatility is a judgmental factor based on the volatility of guideline publicly traded stocks. According to the study, the appropriate DLOM for a closely held stock with a two-year required holding period and a volatility of between 60 and 90 percent is between 28 and 41 percent.

Longstaff Study: Francis Longstaff conducted a study that relies on stock options to estimate the DLOM. While Chaffe based his study on avoiding losses, Longstaff based his study on unrealized gains. Another difference between the two studies is that Longstaff provides an estimate for the upper limit on the value for marketability. The Longstaff study is based on the price of a hypothetical "lookback" option. Exhibit 5 summarizes the Longstaff study results.

Exhibit 4: WMA Pre-IPO Studies Implied DLOM

Time Period Analyzed	Number of Companies Analyzed	Number of Transactions Analyzed	Standard Mean Price Discount	Trimmed Mean Price Discount	Median Price Discount
1975-78	17	31	34.0%	43.4%	52.5%
1979	9	17	55.6%	56.8%	62.7%
1980-82	58	113	48.0%	51.9%	56.5%
1983	85	214	50.1%	55.2%	60.7%
1984	20	33	43.2%	52.9%	73.1%
1985	18	25	41.3%	47.3%	42.6%
1986	47	74	38.5%	44.7%	47.4%
1987	25	40	36.9%	44.9%	43.8%
1988	13	19	41.5%	42.5%	51.8%
1989	9	19	47.3%	46.9%	50.3%
1990	17	23	30.5%	33.0%	48.5%
1991	27	34	24.2%	28.9%	31.8%
1992	36	75	41.9%	47.0%	51.7%
1993	51	110	46.9%	49.9%	53.3%
1994	31	48	31.9%	38.4%	42.0%
1995	42	66	32.2%	47.4%	58.7%
1996	17	22	31.5%	34.5%	44.3%
1997	34	44	28.4%	30.5%	35.2%
1998	14	21	35.0%	39.8%	49.4%
1999	22	28	26.4%	27.1%	27.7%
2000	13	15	18.0%	22.9%	31.9%

For a five-year holding period and 30 percent standard deviation, the implied DLOM is more than 65 percent. Longstaff analyzed securities with a volatility of between 10 and 30 percent because “this range of volatility is consistent with typical stock return volatilities.”¹

Finnerty Study: John Finnerty conducted an option-pricing study that tests the relative importance of transfer restrictions on the one hand and information and equity ownership concentration effects on the other in explaining private-placement discounts, which is an extension of the Longstaff study. Unlike Longstaff, Finnerty did not assume that investors have perfect market-timing ability. Instead, Finnerty modeled the DLOM as the value of an average strike-put option.

In addition to analyzing stock options, Finnerty analyzed 101 restricted stock private placements that occurred between Jan. 1, 1991, and Feb. 3, 1997. The Finnerty private placement study concluded price discounts of 20.13 and 18.41 percent for the day prior to the private placement and for 10 days prior to the private placement, respectively.

Long-Term Equity Anticipation Securities (LEAPS) Studies: In September 2003, Robert Trout published a study analyzing LEAPS and DLOM. Ronald Seaman updated the Trout LEAPS study several times (the most recent update was in September 2013). A LEAP is essentially a long-term stock option that offers price protection for up to two years into the future. Therefore, an investor who desires protection against stock price declines can purchase a LEAPS put option.

The LEAPS studies examined the cost of buying LEAPS put options and concluded that the cost of the LEAPS put option divided by the stock price indicates the DLOM. Trout examined nine LEAPS as of March 2003 with options expiring in January 2005 that were for large companies with actively traded securities. According to Trout, “The data concerning the relative cost of puts as an insurance premium indicate an insurance premium cost equal to about 24 percent of the price. This finding suggests that the minimum discount that one should assign for the lack of marketability of holding privately held stock is at least 24 percent.”²

The 2013 Seaman study updated and extended the Trout study through November 2012. The Seaman study considered the relationship between the price of the LEAPS (*i.e.*, the price discount) and the following variables: (1) company size; (2) company risk; (3) latest year profit margins; (4) latest year return on equity; and (5) company industry. The LEAPS studies concluded that the observed DLOMs were minimum price discounts when applied to the value of closely held companies.

The OPM studies indicate similar price discounts to the above-described empirical studies. In the Chaffe, Longstaff and Finnerty studies, the appropriate DLOM for a closely held company (given certain volatility assumptions) reaches 65 percent. In the LEAPS studies, the price discount is much lower, but the study authors concluded that the indicated price discount represents a minimum DLOM.

1 Francis A. Longstaff, “How Much Can Marketability Affect Security Values?,” *The Journal of Finance* (December 1995), 1771.

2 Robert R. Trout, “Minimum Marketability Discounts,” *Business Valuation Review* (September 2003), 124-25.

DCF Models

The DCF method is based on the principle that value equals the present value of future income. Developed by Christopher Mercer, the quantitative marketability discount model (QMDM) is a shareholder-level DCF model that uses a quantitative analysis to calculate the DLOM.³ The QMDM is based on the following factors: (1) the expected growth rate in subject company value; (2) the expected interim cash flow; (3) the expected holding period; and (4) the required holding period return.

In the QMDM, the valuation analyst values the subject company at the entity level, resulting in a marketable security value. Next, the analyst estimates shareholder value (the non-marketable security value). To calculate the shareholder value, the analyst (1) increases the company’s value by the growth rate during the expected holding period, (2) discounts the future company value using the required holding period return, and (3) adds the present value of the dividend stream received during the holding period to this present value. The calculation of one minus the ratio of shareholder value to entity value equals the DLOM.

In contrast, David Tabak created a DCF model to estimate the DLOM based on the capital-asset-pricing model. This model “focuses on the extra risks imposed on the owner of a security or interest in a business enterprise, and not on the lack of access to capital. In brief, the theory uses market data on the additional return that investors require in order to hold a risky asset, measured by the equity risk premium, to extrapolate the extra return that the holder of an illiquid asset would require.”⁴

Transferability Restrictions

Many empirical studies indicate that company size, block size and dividends affect the DLOM. Other factors that affect DLOM include contractual restrictions, such as a shareholder agreement, right of first refusal, buy/sell agreement and the like. Contractual restrictions can severely limit the marketability of closely held securities. The following list presents some contractual restrictions that may affect the DLOM:

3 Z. Christopher Mercer, *Quantifying Marketability Discounts* (Peabody Publishing 1997).

4 David Tabak, “A CAPM-Based Approach to Calculating Illiquidity Discounts,” *NERA Economic Consulting* publication (Nov. 11, 2002), available at nera.com.

Exhibit 5: Longstaff Study Upper Bounds for Implied DLOM

Marketability Restriction Period	Standard Deviation = 10%	Standard Deviation = 20%	Standard Deviation = 30%
1 Day	0.421	0.844	1.268
5 Days	0.944	1.894	2.852
10 Days	1.337	2.688	4.052
20 Days	1.894	3.817	5.768
30 Days	2.324	4.691	7.100
60 Days	3.299	6.683	10.153
90 Days	4.052	8.232	12.542
180 Days	5.768	11.793	18.082
1 Year	8.232	16.984	26.276
2 Years	11.793	24.643	38.605
5 Years	19.128	40.979	65.772

(1) buy/sell agreements; (2) shareholder or partnership agreements; (3) rights of first refusal; and (4) other contractual transferability restrictions. The more restrictive the agreement or provision, the greater the appropriate DLOM, all else being equal.

Conclusion

Analysts value noncontrolling securities in closely held companies for various bankruptcy purposes. Depending on the valuation approaches and methods that are applied, as well as the benchmark empirical data being used, the analyses might initially conclude the security value on a marketable (as if traded on a stock exchange) basis. In such instances, valuation analysts might apply a valuation adjustment to conclude the final value. This article summarized the factors to consider to measure the DLOM associated with noncontrolling securities of a closely held company. [abi](#)

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